

# Thetafunktionen positiv definiter quadratischer Formen III

## Thetafunktionen mit Charakteristik

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# Wiederholung

## Definition (Gerade Matrix)

$r \in \mathbb{N}$ ;  $A \in M_{r,r}(\mathbb{Z})$  symmetrisch  $\rightarrow$   
 $A$  gerade  $:\Leftrightarrow \forall i \in \{1, \dots, r\} : A_{ii}$  gerade

## Definition (PETERSSONScher Strichoperator)

$k \in \mathbb{Z}$ ;  $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2,2}(\mathbb{R})$ ;  $f : \mathbb{H} \rightarrow \mathbb{C}$  holomorph  $\rightarrow$

$$(f|_k(\gamma))(\tau) := \frac{\det(\gamma)^{\frac{k}{2}}}{(c\tau + d)^k} f(\gamma.\tau) \quad \text{mit} \quad \gamma.\tau := \frac{a\tau + b}{c\tau + d}$$

### Definition ( $A$ -assoziierte Formen)

$r \in \mathbb{N}$ ;  $A \in M_{r,r}(\mathbb{Z})$  gerade  $\rightarrow$

Assoziierte quadratische Form:  $Q : \mathbb{R}^r \rightarrow \mathbb{R}$ ,  $Q(x) := \frac{1}{2}x^T Ax$

Assoziierte Bilinearform:  $Q : \mathbb{R}^r \times \mathbb{R}^r \rightarrow \mathbb{R}$ ,  $B(x, y) := x^T Ay$

$$Q_A^* := Q^*(x) := Q_{A^{-1}}(x)$$

### Definition ( $A$ -assoziierte Thetafunktion)

$r \in \mathbb{N}$ ;  $A \in M_{r,r}(\mathbb{Z})$  gerade, positiv definit  $\rightarrow$

$$\forall \tau \in \mathbb{H} : q := \exp(2\pi i \tau) : \Theta_A(\tau) := \sum_{n \in \mathbb{Z}^r} q^{Q(n)}$$

# Definition

Definition ( $A$ -assoziierte Thetafunktion mit Charakteristik  $\lambda$ )

$r \in \mathbb{N}$ ;  $A \in M_{r,r}(\mathbb{Z})$  gerade, positiv definit;  $\lambda \in A^{-1}\mathbb{Z}^r \rightarrow$

$$\forall \tau \in \mathbb{H} : q := \exp(2\pi i\tau) : \Theta_{A,\lambda}(\tau) := \sum_{n \in \lambda + \mathbb{Z}^r} q^{Q(n)}$$

# Bemerkungen

Bemerkung (Bezug zur alten Definition)

$$\Theta_{A,0} = \Theta_A$$

Beweis.

$$\Theta_{A,0}(\tau) = \sum_{n \in 0 + \mathbb{Z}^r} q^{Q(n)} = \sum_{n \in \mathbb{Z}^r} q^{Q(n)} = \Theta_A(\tau) \quad \square$$

### Bemerkung (Reduktion von $\lambda$ )

$$\forall \mu \in \mathbb{Z}^r : \Theta_{A, \lambda + \mu} = \Theta_{A, \lambda}$$

→ Es reicht  $\lambda \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r$  zu betrachten.

### Beweis.

$$\mu + \mathbb{Z}^r = \mathbb{Z}^r \Rightarrow$$

$$\Theta_{A, \lambda + \mu}(\tau) = \sum_{n \in \lambda + \mu + \mathbb{Z}^r} q^{Q(n)} = \sum_{n \in \lambda + \mathbb{Z}^r} q^{Q(n)} = \Theta_{A, \lambda}(\tau) \quad \square$$

### Bemerkung (Lineare Abhängigkeit)

$$\forall \lambda \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r : \Theta_{A,-\lambda} = \Theta_{A,\lambda}$$

→  $V(A) := \{\Theta_{A,\lambda} | \lambda \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r\}$  ist linear abhängig

### Beweis.

$$\forall x \in \mathbb{R}^r : Q(-x) = Q(x) \wedge -\mathbb{Z}^r = \mathbb{Z}^r \Rightarrow$$

$$\begin{aligned} \Theta_{A,-\lambda}(\tau) &= \sum_{n \in -\lambda + \mathbb{Z}^r} q^{Q(n)} = \sum_{n \in -(-\lambda + \mathbb{Z}^r) = \lambda - \mathbb{Z}^r} q^{Q(-n)} \\ &= \sum_{n \in \lambda + \mathbb{Z}^r} q^{Q(-n)} = \sum_{n \in \lambda + \mathbb{Z}^r} q^{Q(n)} = \Theta_{A,\lambda}(\tau) \quad \square \end{aligned}$$

# Transformationsformeln

## Theorem (Transformationsformeln)

$r \in \mathbb{N}$ ;  $A \in M_{r,r}(\mathbb{Z})$  gerade, positiv definit;  $\lambda \in A^{-1}\mathbb{Z}^r \Rightarrow$

$$\Theta_{A,\lambda}(\tau + 1) = \exp(2\pi i Q(\lambda)) \Theta_{A,\lambda}(\tau) \quad (\text{i})$$

$$\Theta_{A,\lambda} \left( -\frac{1}{\tau} \right) = \frac{(-i\tau)^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{\mu \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r} \exp(2\pi i B(\lambda, \mu)) \Theta_{A,\mu}(\tau) \quad (\text{ii})$$



## Lemmata

In vorherigen Vorträgen schon gezeigt:

## Lemma (1)

\*Theorem  $\Rightarrow f(x) := \exp\left(-\frac{2\pi Q(x)}{t}\right) \rightarrow$

$$(\mathcal{F}f)(x) = \frac{t^{\frac{r}{2}}}{\sqrt{\det(A)}} \exp(-2\pi t Q^*(x))$$

## Lemma (2)

$$f \in \mathcal{S}(\mathbb{R}^r) \Rightarrow \sum_{n \in \mathbb{Z}^r} f(n) = \sum_{n \in \mathbb{Z}^r} (\mathcal{F}f)(n)$$

## Beweis (i)

1.  $\forall n \in \lambda + \mathbb{Z}^r : \exists m \in \mathbb{Z}^r : n = \lambda + m$
2.  $\star(A \text{ symm.}) \wedge 1 \Rightarrow Q(n) = Q(\lambda + m) = Q(\lambda) + B(\lambda, m) + Q(m)$
3.  $\star(A \text{ gerade}) \Rightarrow Q(\mathbb{Z}^r) \subset \mathbb{Z}$
4.  $\star(\lambda \in A^{-1}\mathbb{Z}^r) \Rightarrow A\lambda \in \mathbb{Z}^r \Rightarrow B(\mathbb{Z}^r, \lambda) = \mathbb{Z}^r \cdot A\lambda \in \mathbb{Z}$
5.  $2 \wedge 3 \wedge 4 \Rightarrow \forall n \in \lambda + \mathbb{Z}^r : \exp(2\pi i Q(n)) = \exp(2\pi i Q(\lambda))$
- 6.

$$\begin{aligned}
 \Theta_{A,\lambda}(\tau + 1) &= \sum_{n \in \lambda + \mathbb{Z}^r} \exp(2\pi i(\tau + 1)Q(n)) \\
 &= \sum_{n \in \lambda + \mathbb{Z}^r} \exp(2\pi i\tau Q(n)) \exp(2\pi iQ(n)) \\
 &\stackrel{5}{=} \exp(2\pi iQ(\lambda)) \Theta_{A,\lambda}(\tau)
 \end{aligned}$$

## Beweis (ii)

$$g(x) := f(x + \lambda) \in \mathcal{S}(\mathbb{R}^r)$$

8.

$$\begin{aligned}
 (\mathcal{F}g)(x) &= \int_{\mathbb{R}^r} f(t + \lambda) \exp(-2\pi i x t) dt & \Big| & \quad t \mapsto t - \lambda \\
 &= \int_{\mathbb{R}^r} f(t) \exp(-2\pi i x (t - \lambda)) dt \\
 &= \exp(2\pi i \lambda x) (\mathcal{F}f)(x) & \Big| & \quad \text{Lemma(1)} \\
 &= \frac{t^{\frac{r}{2}}}{\sqrt{\det(A)}} \exp(-2\pi t Q^*(x) + 2\pi i \lambda x)
 \end{aligned}$$

9.  $t \in \mathbb{R}_{\neq 0}^+ \Rightarrow$

$$\begin{aligned} \Theta_{A,\lambda} \left( \frac{i}{t} \right) &= \sum_{n \in \lambda + \mathbb{Z}^r} \exp \left( -\frac{2\pi Q(n)}{t} \right) \stackrel{1}{=} \sum_{m \in \mathbb{Z}^r} \exp \left( -\frac{2\pi Q(m + \lambda)}{t} \right) \\ &= \sum_{m \in \mathbb{Z}^r} g(m) \stackrel{L(2)}{=} \sum_{m \in \mathbb{Z}^r} (\mathcal{F}g)(m) \\ &\stackrel{8}{=} \frac{t^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{m \in \mathbb{Z}^r} \exp(-2\pi t Q^*(m) + 2\pi i \lambda m) \end{aligned}$$

10.  $9 \wedge m = An \Rightarrow$

$$\Theta_{A,\lambda} \left( \frac{i}{t} \right) = \frac{t^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{n \in A^{-1}\mathbb{Z}^r} \exp(-2\pi t Q(n) + 2\pi i B(\lambda, n))$$

11.  $10 \wedge \left[ \mathbb{R}_{\neq 0}^+ \ni t = -i\tau \rightarrow \tau = it \in i\mathbb{R}_{\neq 0}^+ \right] \wedge i\mathbb{R}_{\neq 0}^+$  hat HP  $\Rightarrow$   
 $\forall \tau \in \mathbb{H} :$

$$\Theta_{A,\lambda} \left( -\frac{1}{\tau} \right) = \frac{(-i\tau)^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{n \in A^{-1}\mathbb{Z}^r} \exp(2\pi i Q(n)\tau + 2\pi i B(\lambda, n))$$

12.  $11 \wedge \forall m \in \mathbb{Z}^r : \exp(2\pi i B(\lambda, \mu + m)) = \exp(2\pi i B(\lambda, \mu)) \Rightarrow$

$$\begin{aligned} \Theta_{A,\lambda} \left( -\frac{1}{\tau} \right) &= \frac{(-i\tau)^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{\mu \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r} \sum_{n \in \mu + \mathbb{Z}^r} \exp(2\pi i Q(n)\tau + 2\pi i B(\lambda, \mu)) \\ &= \frac{(-i\tau)^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{\mu \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r} \exp(2\pi i B(\lambda, \mu)) \Theta_{A,\mu}(\tau) \quad \square \end{aligned}$$

# Bemerkung

## Bemerkung (Erweiterung auf $\Gamma_1$ )

$r \in \mathbb{N}$  gerade;  $A \in M_{r,r}(\mathbb{Z})$  gerade, positiv definit;  $\lambda \in A^{-1}\mathbb{Z}^r \Rightarrow$

$$\forall \gamma \in \Gamma_1 : \Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma) \in \text{span}_{\mathbb{C}} \left( V := \left\{ \Theta_{A,\mu} \mid \mu \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r \right\} \right)$$

## Beweis.

$$\Theta_{A,\lambda}(T.\tau) = \Theta_{A,\lambda}(\tau + 1) \in \text{span}_{\mathbb{C}}(V) \wedge$$

$$\Theta_{A,\lambda}(S.\tau) = \Theta_{A,\lambda}\left(-\frac{1}{\tau}\right) \in \text{span}_{\mathbb{C}}(V) \wedge \Gamma_1 = \langle T, S \rangle \wedge$$

$$| \text{linkslin}ear, \forall \gamma_1, \gamma_2 \in \Gamma_1 : (\Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma_1))|_{\frac{r}{2}}(\gamma_2) = \Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma_1\gamma_2) \Rightarrow$$

$$(\Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma))(\tau) = \frac{1}{(c\tau + d)^{\frac{r}{2}}} \Theta_{A,\lambda}(\gamma.\tau) \in \text{span}_{\mathbb{C}}(V) \quad \square$$

## Hilfsdefinition

## Definition

$A \in M_{r,r}(\mathbb{Z})$  gerade;  $\lambda \in A^{-1}\mathbb{Z}^r$ ;  $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_1$ ;  $c \neq 0 \rightarrow$

$$S_\gamma(\lambda) := \sum_{\mu \in \mathbb{Z}^r / c\mathbb{Z}^r} \exp\left(2\pi i \frac{a}{c} Q(\mu + \lambda)\right)$$

## Bemerkung

$$\forall \mu \in \mathbb{Z}^r / c\mathbb{Z}^r : \exp\left(2\pi i \frac{a}{c} Q(\mu + c\mathbb{Z}^r)\right) = \exp\left(2\pi i \frac{a}{c} Q(\mu)\right)$$

$\Rightarrow S_\gamma(\lambda)$  wohldefiniert (unabhängig von Restklassenwahl)  $\wedge$

$$\forall \lambda \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r : S_\gamma(\lambda + \mathbb{Z}^r) = S_\gamma(\lambda)$$





# Strichformel

## Theorem (Strichformel)

$r \in \mathbb{N}$  gerade;  $A \in M_{r,r}(\mathbb{Z})$  gerade, positiv definit;  $\lambda \in A^{-1}\mathbb{Z}^r$ ;  
 $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_1$ ;  $c \neq 0 \Rightarrow$

$$\Theta_{A,\lambda} \Big|_{\frac{r}{2}}(\gamma) = \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{2}} \sqrt{\det(A)}} \cdot$$

$$\sum_{\lambda' \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r}$$

$$\exp(-2\pi i [bB(\lambda, \lambda') + bdQ(\lambda')]) \cdot$$

$$S_\gamma(\lambda + d\lambda') \cdot \Theta_{A,\lambda'}$$

1.  $S_{-\gamma} = S_{\gamma} \wedge |_{\frac{r}{2}}(-\gamma) = (-1)^{\frac{r}{2}} |_{\frac{r}{2}}(\gamma) \Rightarrow \gamma - > -\gamma \Rightarrow \mathfrak{E} \quad c > 0$
2.  $V_c := \text{diag}(c, 1) \Rightarrow \gamma = \begin{pmatrix} c^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & bc \\ 1 & d \end{pmatrix} \begin{pmatrix} c & 0 \\ 0 & 1 \end{pmatrix} = V_c^{-1} T^a S T^d V_c$
3.  $1 \Rightarrow$

$$\begin{aligned}
 (\Theta_{A,\lambda} |_{\frac{r}{2}}(V_c^{-1}))(\tau) &= \frac{(\det(V_c^{-1}))^{\frac{r}{4}}}{1^k} \Theta_{A,\lambda} \left( \frac{\tau}{c} \right) \\
 &= c^{-\frac{r}{4}} \sum_{n \in \lambda + \mathbb{Z}^r} q^{\frac{Q(n)}{c}} \\
 &= c^{-\frac{r}{4}} \sum_{m \in \frac{\lambda}{c} + \frac{1}{c} \mathbb{Z}^r} q^{cQ(m)} \\
 &= c^{-\frac{r}{4}} \sum_{\mu \in \mathbb{Z}_c^r} \sum_{m \in \frac{\lambda + \mu}{c} + \mathbb{Z}^r} q^{cQ(m)} \\
 &= c^{-\frac{r}{4}} \sum_{\mu \in \mathbb{Z}_c^r} \left( \Theta_{cA, \frac{\lambda + \mu}{c}} \right) (\tau)
 \end{aligned}$$

4.  $\star(A) \wedge 1 \Rightarrow cA$  positiv definit, gerade  $\wedge$   
 $\forall \mu \in \mathbb{Z}^r : \forall \lambda \in A^{-1}\mathbb{Z}^r : cA \frac{\lambda + \mu}{c} = A\lambda + A\mu \in \mathbb{Z}^r \rightarrow \frac{\lambda + \mu}{c} \in (cA)^{-1}\mathbb{Z}^r$   
 $\Rightarrow$  Transformationsformeln auf  $\Theta_{cA, \frac{\lambda + \mu}{c}}$  anwendbar  $\Rightarrow$

$$\begin{aligned} \Theta_{cA, \frac{\lambda + \mu}{c}} \Big|_{\frac{r}{2}} (T^a) &\stackrel{T(i)}{=} \exp \left( 2\pi i a c Q \left( \frac{\lambda + \mu}{c} \right) \right) \Theta_{cA, \frac{\lambda + \mu}{c}} \\ &= \exp \left( 2\pi i \frac{a}{c} Q(\lambda + \mu) \right) \Theta_{cA, \frac{\lambda + \mu}{c}} \end{aligned}$$

5.

$$\begin{aligned} \Theta_{cA, \frac{\lambda + \mu}{c}} \Big|_{\frac{r}{2}} (T^a S) &\stackrel{4}{=} \exp \left( 2\pi i \frac{a}{c} Q(\lambda + \mu) \right) \Theta_{cA, \frac{\lambda + \mu}{c}} \Big|_{\frac{r}{2}} (S) \stackrel{T(ii)}{=} \\ \exp(\dots) &\frac{(-i)^{\frac{r}{2}}}{\sqrt{\det(cA)}} \sum_{\nu \in (cA)^{-1}\mathbb{Z}^r / \mathbb{Z}^r} \exp(2\pi i B(\lambda + \mu, \nu)) \Theta_{cA, \nu} \end{aligned}$$

6.

$$\begin{aligned}
 & \Theta_{cA, \frac{\lambda+\mu}{c}} \Big|_{\frac{r}{2}} \left( T^a S T^d \right) \stackrel{5}{=} \\
 & \exp(\dots) \frac{(-i)^{\frac{r}{2}}}{\sqrt{\det(cA)}} \sum_{\nu \in (cA)^{-1} \mathbb{Z}^r / \mathbb{Z}^r} \exp(2\pi i B(\lambda + \mu, \nu)) \Theta_{cA, \nu} \Big|_{\frac{r}{2}} (T^d) \stackrel{T(i)}{=} \\
 & \exp(\dots) \frac{(-i)^{\frac{r}{2}}}{\sqrt{\det(cA)}} \sum_{\nu \in (cA)^{-1} \mathbb{Z}^r / \mathbb{Z}^r} \exp(2\pi i [B(\lambda + \mu, \nu) + cdQ(\nu)]) \Theta_{cA, \nu} = \\
 & \frac{(-i)^{\frac{r}{2}}}{\sqrt{\det(cA)}} \sum_{\nu \in (cA)^{-1} \mathbb{Z}^r / \mathbb{Z}^r} \exp \left( 2\pi i \left[ \frac{a}{c} Q(\lambda + \mu) + B(\lambda + \mu, \nu) + cdQ(\nu) \right] \right) \Theta_{cA, \nu}
 \end{aligned}$$

7.

$$\Theta_{cA, \lambda} \Big|_{\frac{r}{2}} \left( V_c^{-1} T^a S T^d \right) \stackrel{3}{=} \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{4}} \sqrt{\det(cA)}} \cdot \sum_{\substack{\mu \in \mathbb{Z}_c^r \\ \nu \in (cA)^{-1} \mathbb{Z}^r / \mathbb{Z}^r}} \exp \left( 2\pi i \left[ \frac{a}{c} Q(\lambda + \mu) + B(\lambda + \mu, \nu) + cdQ(\nu) \right] \right) \Theta_{cA, \nu}$$

8.  $7 \wedge \forall \nu \in (cA)^{-1} \mathbb{Z}^r / \mathbb{Z}^r : \exists \lambda' \in A^{-1} \mathbb{Z}^r / \mathbb{Z}^r : \exists \mu' \in \mathbb{Z}_c^r : \nu = \frac{\lambda' + \mu'}{c} \Rightarrow$ 

$$\Theta_{cA, \lambda} \Big|_{\frac{r}{2}} \left( V_c^{-1} T^a S T^d \right) = \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{4}} \sqrt{\det(cA)}} \sum_{\substack{\mu, \mu' \in \mathbb{Z}_c^r \\ \lambda' \in A^{-1} \mathbb{Z}^r / \mathbb{Z}^r}} \exp \left( 2\pi i \left[ \frac{a}{c} Q(\lambda + \mu) + \frac{1}{c} B(\lambda + \mu, \lambda' + \mu') + \frac{d}{c} Q(\lambda' + \mu') \right] \right) \Theta_{cA, \frac{\lambda' + \mu'}{c}}$$

$$9. \quad x := \lambda + \mu, y := \lambda' + \mu' \wedge$$

$$\frac{a}{c}Q(x) + \frac{1}{c}B(x, y) + \frac{d}{c}Q(y) = \frac{a}{c}Q(x + dy) - bB(x, y) - bdQ(y) \wedge$$

$$B(\lambda + \mu, \lambda' + \mu') - B(\lambda, \lambda') = B(\lambda, \lambda') + B(\lambda', \mu) + B(\mu, \mu') \in \mathbb{Z} \wedge$$

$$Q(\lambda' + \mu') - Q(\lambda') = B(\lambda', \mu') + Q(\mu') \in \mathbb{Z} \Rightarrow$$

$$\exp \left( 2\pi i \left[ \frac{a}{c}Q(\lambda + \mu) + \frac{1}{c}B(\lambda + \mu, \lambda' + \mu') + \frac{d}{c}Q(\lambda' + \mu') \right] \right) =$$

$$\exp \left( 2\pi i \left[ \frac{a}{c}Q(\lambda + \mu + d\lambda' + d\mu') - bB(\lambda, \lambda') - bdQ(\lambda') \right] \right)$$

$$10. \quad 8 \wedge 9 \wedge \mu \mapsto \mu - d\mu' \Rightarrow$$

$$\Theta_{cA, \lambda} \Big|_{\frac{r}{2}} \left( V_c^{-1} T^a S T^d \right) = \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{4}} \sqrt{\det(cA)}} \cdot \sum_{\substack{\mu, \mu' \in \mathbb{Z}_c^r \\ \lambda' \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r}}$$

$$\exp \left( 2\pi i \left[ \frac{a}{c}Q(\lambda + \mu + d\lambda') - bB(\lambda, \lambda') - bdQ(\lambda') \right] \right) \Theta_{cA, \frac{\lambda' + \mu'}{c}}$$

11.

$$\begin{aligned}
& \Theta_{cA, \lambda} \Big|_{\frac{r}{2}} \left( V_c^{-1} T^a S T^d \right) \\
& \stackrel{10}{=} \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{4}} \sqrt{\det(cA)}} \frac{c^{-\frac{r}{4}}}{c^{-\frac{r}{4}}} \sum_{\substack{\mu \in \mathbb{Z}_c^r \\ \lambda' \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r}} \\
& \exp \left( 2\pi i \left[ \frac{a}{c} Q(\lambda + \mu + d\lambda') - bB(\lambda, \lambda') - bdQ(\lambda') \right] \right) \sum_{\mu' \in \mathbb{Z}_c^r} \Theta_{cA, \frac{\lambda' + \mu'}{c}} \\
& \stackrel{3}{=} \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{2}} \sqrt{\det(A)}} \sum_{\substack{\mu \in \mathbb{Z}_c^r \\ \lambda' \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r}} \\
& \exp \left( 2\pi i \left[ \frac{a}{c} Q(\lambda + \mu + d\lambda') - bB(\lambda, \lambda') - bdQ(\lambda') \right] \right) \Theta_{A, \lambda'} \Big|_{\frac{r}{2}} \left( V_c^{-1} \right)
\end{aligned}$$

12.

$$\begin{aligned}
& \Theta_{cA, \lambda} \Big|_{\frac{r}{2}} \left( V_c^{-1} T^a S T^d \right) \\
& \stackrel{11}{=} \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{2}} \sqrt{\det(A)}} \sum_{\lambda' \in A^{-1} \mathbb{Z}^r / \mathbb{Z}^r} \exp(-2\pi i [bB(\lambda, \lambda') + bdQ(\lambda')]) \cdot \\
& \quad \left[ \sum_{\mu \in \mathbb{Z}_c^r} \exp\left(\frac{a}{c} Q(\mu + \lambda + d\lambda')\right) \right] \Theta_{A, \lambda'} \Big|_{\frac{r}{2}} \left( V_c^{-1} \right) \\
& = \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{2}} \sqrt{\det(A)}} \sum_{\lambda' \in A^{-1} \mathbb{Z}^r / \mathbb{Z}^r} \exp(-2\pi i [bB(\lambda, \lambda') + bdQ(\lambda')]) \cdot \\
& \quad S_\gamma(\lambda + d\lambda') \Theta_{A, \lambda'} \Big|_{\frac{r}{2}} \left( V_c^{-1} \right)
\end{aligned}$$

$$13. \quad 2, 11 \Rightarrow \Theta_{A, \lambda} \Big|_{\frac{r}{2}}(\gamma) = \Theta_{cA, \frac{\lambda + \mu}{c}} \Big|_{\frac{r}{2}} \left( V_c^{-1} T^a S T^d V_c \right)$$

□



Ende

Vielen Dank für Ihre Aufmerksamkeit!