Anonymity Networks

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27th July 2017

In the lecture *'Information Theory and Statistical Physics'* by Prof. Dr. Johannes Berg

motivation

motivation

- hide initiator of a message in a computer network

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- hide initiator of a message in a computer network
- safe whistleblowing under corporate and state surveillance

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- ‘deniable communication’

motivation

- hide initiator of a message in a computer network
- safe whistleblowing under corporate and state surveillance
- ‘deniable communication’
- decentralized

idea

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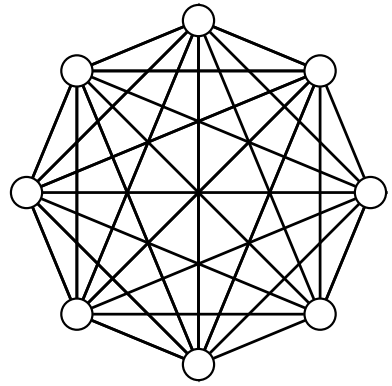
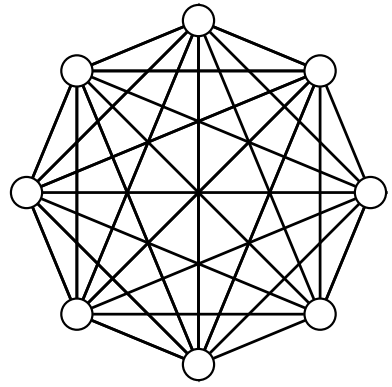


node network participant

link possible message path

idea



node network participant

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- all nodes have equal weight

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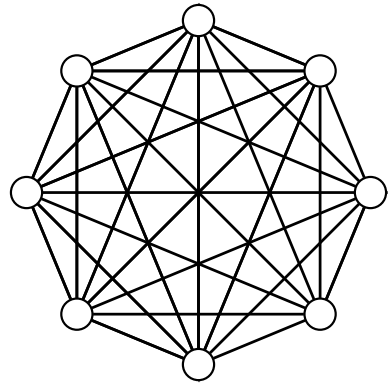

node network participant

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- all nodes have equal weight
- message unmodifiable, only receiver is known

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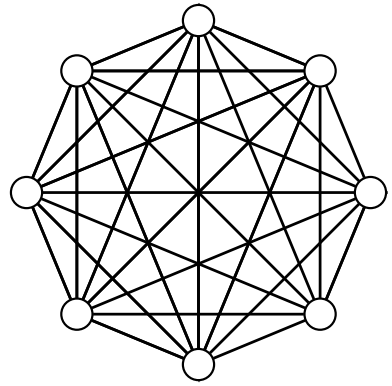

node network participant

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- each node on path: biased coin flip: forward or deliver

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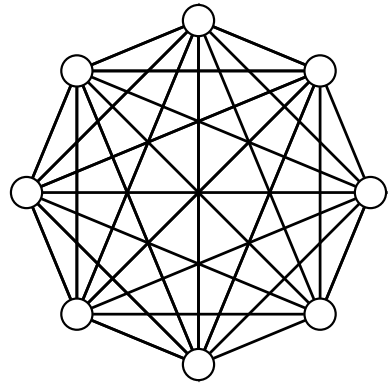

node network participant

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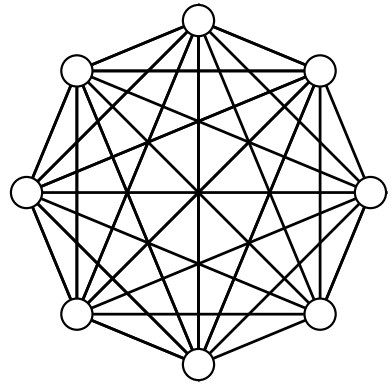

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- all nodes have equal weight
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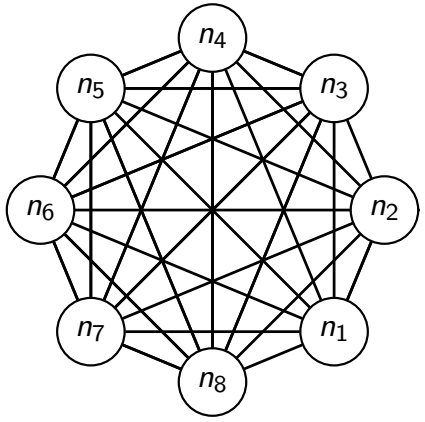
$\rightarrow$ message initator gets lost in the crowd

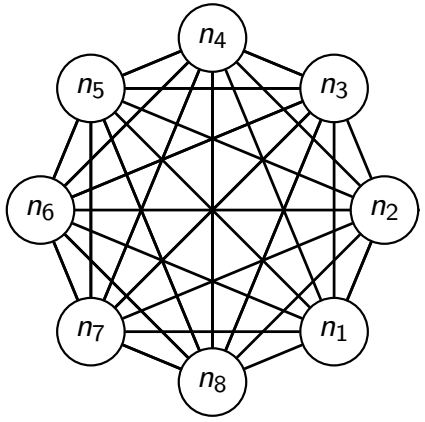
model

model



- N nodes $n_1, \ldots, n_N$ with $\mathbb{P}(n_i \text{ is initiator}) =: \mathbb{P}(X = n_i) =: p_i$

model



- N nodes $n_1, \ldots, n_N$ with $\mathbb{P}(n_i \text{ is initiator}) =: \mathbb{P}(X = n_i) =: p_i$
- n_i probably innocent $\leftrightarrow p_i \leq \frac{1}{2}$

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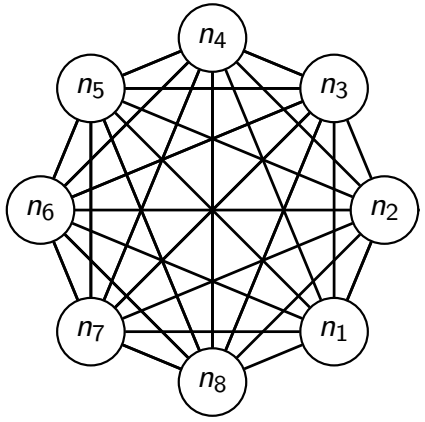

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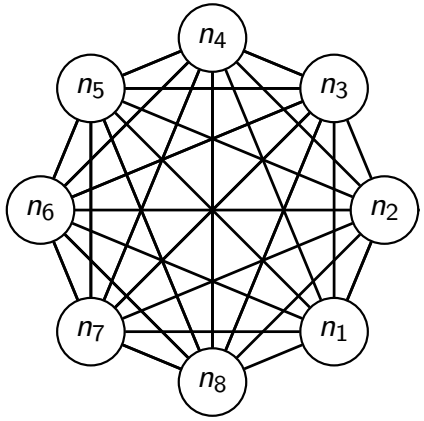

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- n_i probably innocent $\leftrightarrow p_i \leq \frac{1}{2}$
- forwarding probability λ

if message received **then**
    flip biased coin $\mathbb{P}(\text{heads}) = \lambda$
    if heads **then**
        forward to a uniformly chosen node
    else
        deliver to receiver
    end if
end if

degree of anonymity

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best case $\overline{X} := X : \forall i \in \{1, \ldots, N\} : p_i = \frac{1}{N}$

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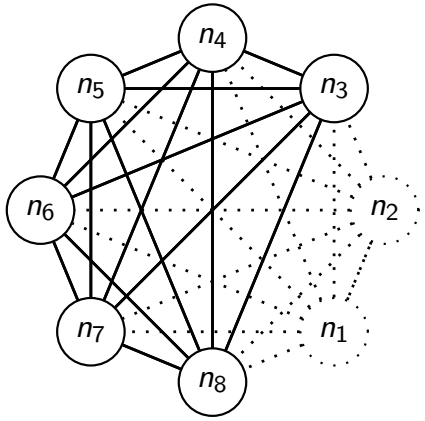
$$\boxed{d(X) := 1 - \frac{\overline{H} - H(X)}{\overline{H}} = \frac{H(X)}{\overline{H}} \in [0, 1]}$$

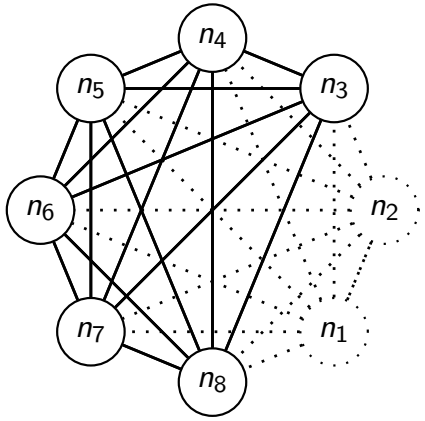
corruption

corruption



- $0 \leq C < N$ corrupt nodes (incoming message passer known)

corruption



- $0 \leq C < N$ corrupt nodes (incoming message passer known)
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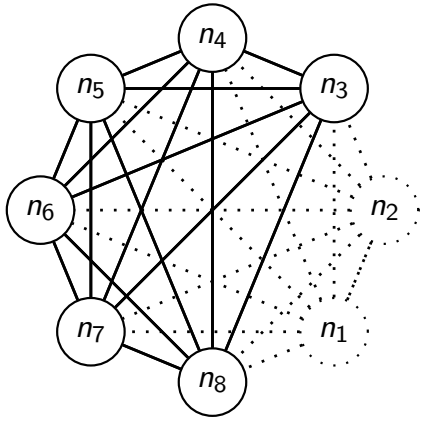
corruption



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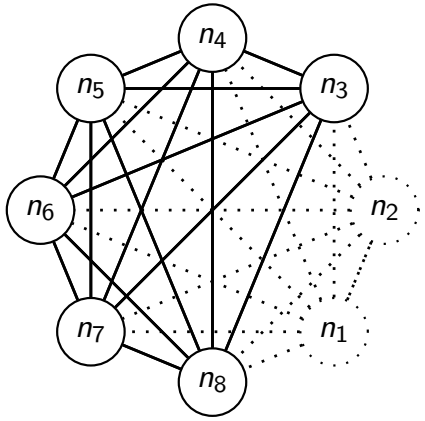

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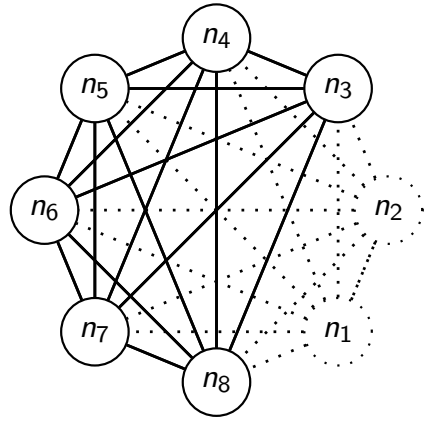


- $0 \leq C < N$ corrupt nodes (incoming message passer known)
- behave normally
- wait for message to be passed to us
- analyze probability of passer being initiator

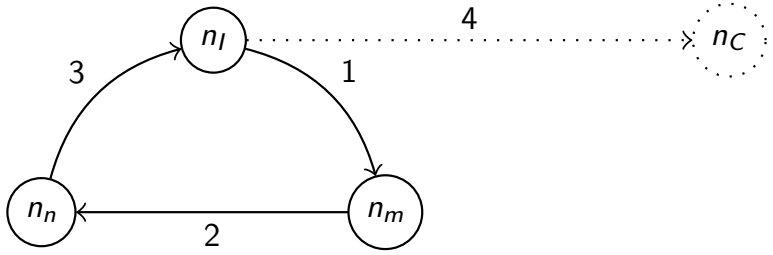
$\mathbb{P}(\text{passer is initiator}) > \frac{1}{2} \rightarrow$ unmasked

analysis

events

analysis

events



analysis

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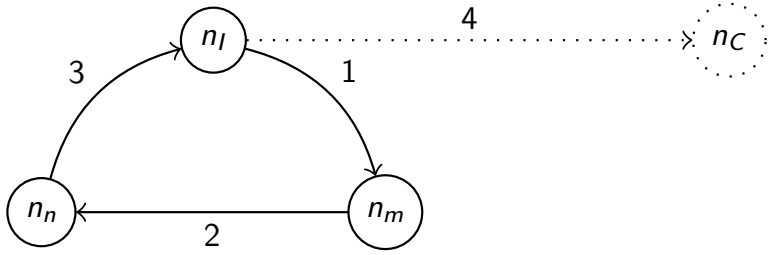
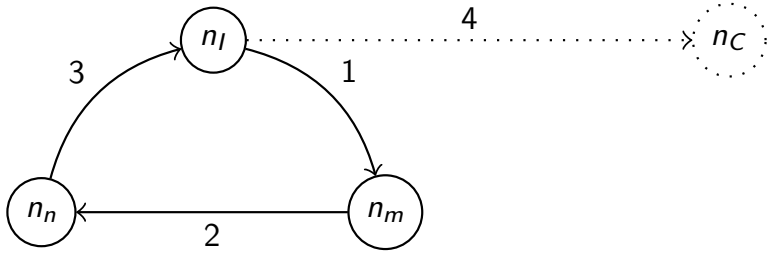


let $k > 0$

analysis

events

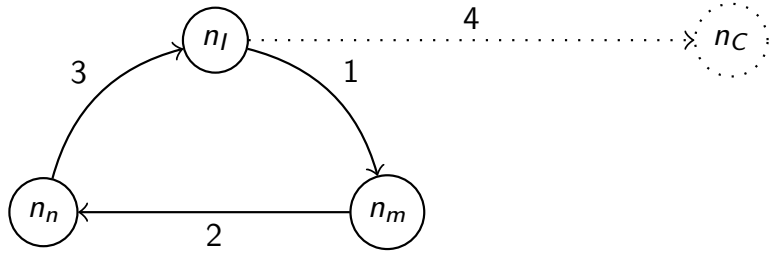


let $k > 0$

$H_k :=$ first corrupt node is at the kth path-position

analysis

events



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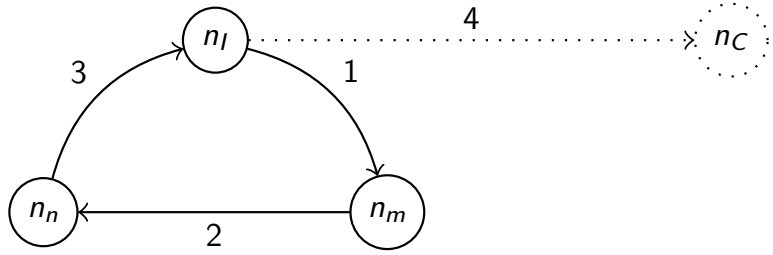

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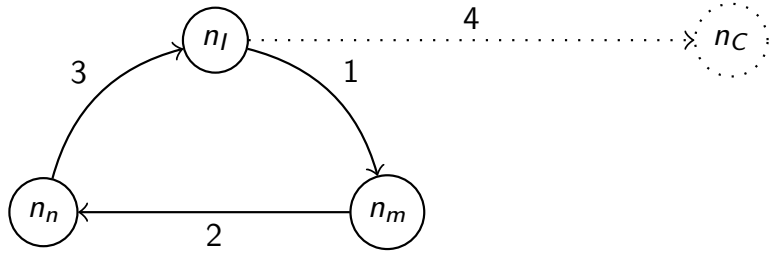

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$$\boxed{\mathbb{P}(\text{passer is initiator}) = \mathbb{P}(I|H_{1+})}$$

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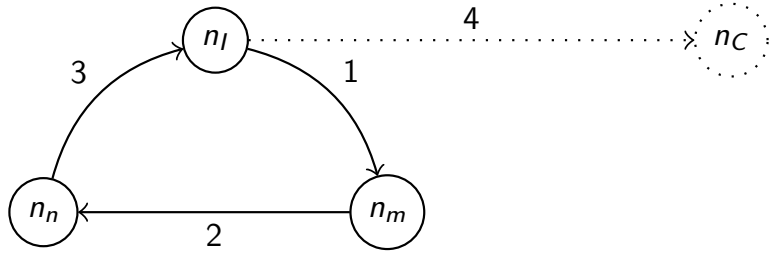

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note: $H_1 \to I$, but $I \not\to H_1$

analysis

general probability I

$$\boxed{\mathbb{P}(I|H_{1+}) = \frac{N - \lambda(N - C - 1)}{N}}$$

analysis

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proof:

analysis

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$$\mathbb{P}(H_k) = \left(\lambda \cdot \frac{N - C}{N}\right)^{k-1} \cdot \left(\lambda \cdot \frac{C}{N}\right)$$

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$$H_1 \to I \Rightarrow \mathbb{P}(I|H_1) = 1$$

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$$\mathbb{P}(I|H_{2+}) = \frac{1}{N - C}$$

analysis

general probability II

$$\mathbb{P}(I) \overset{TP}{=} \mathbb{P}(H_1)\,\mathbb{P}(I|H_1) + \mathbb{P}(H_{2+})\,\mathbb{P}(I|H_{2+})$$

analysis

general probability II

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$$\mathbb{P}(I|H_{1+}) \overset{CP}{=} \frac{\mathbb{P}(I \wedge H_{1+})}{\mathbb{P}(H_{1+})}$$

analysis

general probability II

$$
\begin{aligned}
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&= \frac{\lambda \cdot C}{N} \cdot \left(1 + \frac{\lambda}{N - \lambda \cdot (N - C)}\right) \\
\mathbb{P}(I|H_{1+}) &\overset{CP}{=} \frac{\mathbb{P}(I \wedge H_{1+})}{\mathbb{P}(H_{1+})} \quad | \quad I \to H_{1+} \\
&= \frac{\mathbb{P}(I)}{\mathbb{P}(H_{1+})}
\end{aligned}
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\end{aligned}$$

□

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□

good node $\mathbb{P}(\text{good node i is initiator}) = \frac{1-\mathbb{P}(I|H_{1+})}{N-C-1} = \frac{\lambda}{N} < \frac{1}{N} \leq \frac{1}{2}$

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□

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$\Rightarrow$ all good nodes besides passer are innocent

analysis

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$\Rightarrow$ all good nodes besides passer are innocent

corrupt node $\mathbb{P}(\text{corrupt node i is initiator}) = 0$

analysis

passer innocence

analysis

passer innocence

$$\text{passer innocent} \Leftrightarrow \lambda > \frac{1}{2} \wedge N \geq \frac{1}{1 - \frac{1}{2 \cdot \lambda}} \cdot (C + 1)$$

analysis

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proof:

analysis

passer innocence

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proof:

$$\frac{1}{2} \geq \mathbb{P}(I | H_{1+})$$

analysis

passer innocence

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proof:

$$\frac{1}{2} \geq \mathbb{P}(I|H_{1+}) = \frac{N - \lambda(N - C - 1)}{N}$$

analysis

passer innocence

$$\boxed{\text{passer innocent} \Leftrightarrow \lambda > \frac{1}{2} \wedge N \geq \frac{1}{1 - \frac{1}{2 \cdot \lambda}} \cdot (C + 1)}$$

proof:

$$\frac{1}{2} \geq \mathbb{P}(I|H_{1+}) = \frac{N - \lambda(N - C - 1)}{N} \quad \Big| \quad (\lambda - \frac{1}{2}) > 0$$

$$\Leftrightarrow N \geq \frac{1}{1 - \frac{1}{2 \cdot \lambda}} \cdot (C + 1)$$

□

analysis

degree of anonymity

analysis

degree of anonymity

$$d(X) = -\frac{C \cdot 0 + \mathbb{P}(I|H_{1+}) \cdot \ln(\mathbb{P}(I|H_{1+})) + (N - C - 1) \cdot \frac{\lambda}{N} \cdot \ln\left(\frac{\lambda}{N}\right)}{\ln(N - C)} =$$

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$$= \ldots = \frac{(N-\lambda\cdot(N-C-1))\cdot\ln\left(\frac{N}{N-\lambda\cdot(N-C-1)}\right)+\lambda\cdot(N-C-1)\cdot\ln\left(\frac{N}{\lambda}\right)}{N\cdot\ln(N-C)}$$

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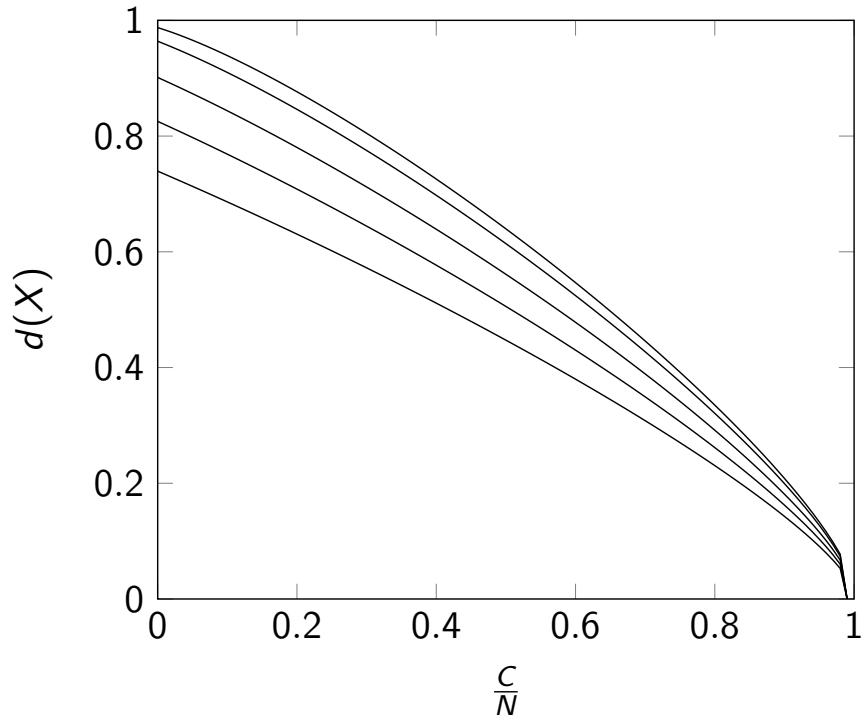

conclusion

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- full paper on `http://frign.de/`