Thetafunktionen positiv definiter quadratischer Formen III

Thetafunktionen mit Charakteristik

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24. Juli 2017

Verwendete Literatur: S. Zwegers, *Lecture Notes on Modular Forms*

Wiederholung

Definition (Gerade Matrix)

$r \in \mathbb{N}$; $A \in M_{r,r}(\mathbb{Z})$ symmetrisch $\rightarrow$
A gerade $:\Leftrightarrow \forall i \in \{1, \ldots, r\} : A_{ii}$ gerade

Definition (PETERSSONscher Strichoperator)

$k \in \mathbb{Z}$; $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in M_{2,2}(\mathbb{R})$; $f : \mathbb{H} \rightarrow \mathbb{C}$ holomorph $\rightarrow$

$$(f|_k(\gamma))(\tau) := \frac{\det(\gamma)^{\frac{k}{2}}}{(c\tau + d)^k} f(\gamma.\tau) \quad \text{mit} \quad \gamma.\tau := \frac{a\tau + b}{c\tau + d}$$

Definition (A-assoziierte Formen)

$r \in \mathbb{N}$; $A \in M_{r,r}(\mathbb{Z})$ gerade $\rightarrow$
Assoziierte quadratische Form: $Q : \mathbb{R}^r \rightarrow \mathbb{R}, Q(x) := \frac{1}{2}x^T Ax$
Assoziierte Bilinearform: $Q : \mathbb{R}^r \times \mathbb{R}^r \rightarrow \mathbb{R}, B(x, y) := x^T Ay$

$Q_A^* := Q^*(x) := Q_{A^{-1}}(x)$

Definition (A-assoziierte Thetafunktion)

$r \in \mathbb{N}$; $A \in M_{r,r}(\mathbb{Z})$ gerade, positiv definit $\rightarrow$

$$\forall \tau \in \mathbb{H} : q := \exp(2\pi i\tau) : \Theta_A(\tau) := \sum_{n\in\mathbb{Z}^r} q^{Q(n)}$$

Definition

Definition (A-assoziierte Thetafunktion mit Charakteristik λ)

$r \in \mathbb{N}$; $A \in M_{r,r}(\mathbb{Z})$ gerade, positiv definit; $\lambda \in A^{-1}\mathbb{Z}^r \rightarrow$

$$\forall \tau \in \mathbb{H} : q := \exp(2\pi i \tau) : \Theta_{A,\lambda}(\tau) := \sum_{n \in \lambda + \mathbb{Z}^r} q^{Q(n)}$$

Bemerkungen

Bemerkung (Bezug zur alten Definition)

$$\Theta_{A,0} = \Theta_A$$

Beweis.

$$\Theta_{A,0}(\tau) = \sum_{n \in 0 + \mathbb{Z}^r} q^{Q(n)} = \sum_{n \in \mathbb{Z}^r} q^{Q(n)} = \Theta_A(\tau) \qquad \square$$

Bemerkung (Reduktion von λ)

$$\forall \mu \in \mathbb{Z}^r : \Theta_{A,\lambda+\mu} = \Theta_{A,\lambda}$$

$\rightarrow$ Es reicht $\lambda \in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r$ zu betrachten.

Beweis.

$\mu + \mathbb{Z}^r = \mathbb{Z}^r \Rightarrow$

$$\Theta_{A,\lambda+\mu}(\tau) = \sum_{n \in \lambda+\mu+\mathbb{Z}^r} q^{Q(n)} = \sum_{n \in \lambda+\mathbb{Z}^r} q^{Q(n)} = \Theta_{A,\lambda}(\tau) \qquad \square$$

Bemerkung (Lineare Abhängigkeit)

$$\forall \lambda \in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r : \Theta_{A,-\lambda} = \Theta_{A,\lambda}$$

$\rightarrow V(A) := \{\Theta_{A,\lambda} | \lambda \in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r\}$ ist linear abhängig

Beweis.

$\forall x \in \mathbb{R}^r : Q(-x) = Q(x) \wedge -\mathbb{Z}^r = \mathbb{Z}^r \Rightarrow$

$$\Theta_{A,-\lambda}(\tau) = \sum_{n \in -\lambda + \mathbb{Z}^r} q^{Q(n)} = \sum_{n \in -(-\lambda+\mathbb{Z}^r) = \lambda - \mathbb{Z}^r} q^{Q(-n)}$$

$$= \sum_{n \in \lambda + \mathbb{Z}^r} q^{Q(-n)} = \sum_{n \in \lambda + \mathbb{Z}^r} q^{Q(n)} = \Theta_{A,\lambda}(\tau) \qquad \square$$

Transformationsformeln

Theorem (Transformationsformeln)

$r \in \mathbb{N}$; $A \in M_{r,r}(\mathbb{Z})$ *gerade, positiv definit;* $\lambda \in A^{-1}\mathbb{Z}^r \Rightarrow$

$$\Theta_{A,\lambda}(\tau + 1) = \exp(2\pi i Q(\lambda))\Theta_{A,\lambda}(\tau) \quad \text{(i)}$$

$$\Theta_{A,\lambda}\left(-\frac{1}{\tau}\right) = \frac{(-i\tau)^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{\mu \in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r} \exp(2\pi i B(\lambda, \mu))\Theta_{A,\mu}(\tau) \quad \text{(ii)}$$

Lemmata

In vorherigen Vorträgen schon gezeigt:

Lemma (1)

$\star_{Theorem} \Rightarrow f(x) := \exp\left(-\frac{2\pi Q(x)}{t}\right) \rightarrow$

$$(\mathcal{F}f)(x) = \frac{t^{\frac{r}{2}}}{\sqrt{\det(A)}} \exp(-2\pi t Q^{\star}(x))$$

Lemma (2)

$$f \in \mathcal{S}(\mathbb{R}^r) \Rightarrow \sum_{n \in \mathbb{Z}^r} f(n) = \sum_{n \in \mathbb{Z}^r} (\mathcal{F}f)(n)$$

Beweis (i)

1. $\forall n \in \lambda + \mathbb{Z}^r : \exists m \in \mathbb{Z}^r : n = \lambda + m$
2. $\star(A \text{ symm.}) \wedge 1 \Rightarrow Q(n) = Q(\lambda + m) = Q(\lambda) + B(\lambda, m) + Q(m)$
3. $\star(A \text{ gerade}) \Rightarrow Q(\mathbb{Z}^r) \subset \mathbb{Z}$
4. $\star(\lambda \in A^{-1}\mathbb{Z}^r) \Rightarrow A\lambda \in \mathbb{Z}^r \Rightarrow B(\mathbb{Z}^r, \lambda) = \mathbb{Z}^r \cdot A\lambda \in \mathbb{Z}$
5. $2 \wedge 3 \wedge 4 \Rightarrow \forall n \in \lambda + \mathbb{Z}^r : \exp(2\pi i Q(n)) = \exp(2\pi i Q(\lambda))$
6. $$\begin{aligned}\Theta_{A,\lambda}(\tau + 1) &= \sum_{n \in \lambda + \mathbb{Z}^r} \exp\left(2\pi i(\tau + 1) Q(n)\right) \\ &= \sum_{n \in \lambda + \mathbb{Z}^r} \exp\left(2\pi i \tau Q(n)\right) \exp\left(2\pi i Q(n)\right) \\ &\overset{5}{=} \exp(2\pi i Q(\lambda)) \Theta_{A,\lambda}(\tau)\end{aligned}$$

Beweis (ii)

$$g(x) := f(x+\lambda) \in \mathcal{S}(\mathbb{R}^r)$$

8.

$$\begin{aligned}
(\mathcal{F}g)(x) &= \int_{\mathbb{R}^r} f(t+\lambda)\exp(-2\pi i x t)\,\mathrm{d}t && \Big| \quad t \mapsto t-\lambda \\
&= \int_{\mathbb{R}^r} f(t)\exp(-2\pi i x (t-\lambda))\,\mathrm{d}t \\
&= \exp(2\pi i \lambda x)(\mathcal{F}f)(x) && \Big| \quad \text{Lemma(1)} \\
&= \frac{t^{\frac{r}{2}}}{\sqrt{\det(A)}}\exp(-2\pi t Q^{\star}(x) + 2\pi i \lambda x)
\end{aligned}$$

9. $t \in \mathbb{R}^+_{\neq 0} \Rightarrow$

$$\Theta_{A,\lambda}\left(\frac{i}{t}\right) = \sum_{n \in \lambda + \mathbb{Z}^r} \exp\left(-\frac{2\pi Q(n)}{t}\right) \stackrel{1}{=} \sum_{m \in \mathbb{Z}^r} \exp\left(-\frac{2\pi Q(m+\lambda)}{t}\right)$$

$$= \sum_{m \in \mathbb{Z}^r} g(m) \stackrel{L(2)}{=} \sum_{m \in \mathbb{Z}^r} (\mathcal{F}g)(m)$$

$$\stackrel{8}{=} \frac{t^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{m \in \mathbb{Z}^r} \exp(-2\pi t Q^\star(m) + 2\pi i \lambda m)$$

10. $9 \wedge m = An \Rightarrow$

$$\Theta_{A,\lambda}\left(\frac{i}{t}\right) = \frac{t^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{n \in A^{-1}\mathbb{Z}^r} \exp(-2\pi t Q(n) + 2\pi i B(\lambda, n))$$

11. $10 \wedge \left[\mathbb{R}^+_{\neq 0} \ni t = -i\tau \to \tau = it \in i\mathbb{R}^+_{\neq 0}\right] \wedge i\mathbb{R}^+_{\neq 0}$ hat HP $\Rightarrow$

    $\forall \tau \in \mathbb{H}$:

$$\Theta_{A,\lambda}\left(-\frac{1}{\tau}\right) = \frac{(-i\tau)^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{n \in A^{-1}\mathbb{Z}^r} \exp(2\pi i Q(n)\tau + 2\pi i B(\lambda, n))$$

12. $11 \wedge \forall m \in \mathbb{Z}^r : \exp(2\pi i B(\lambda, \mu + m)) = \exp(2\pi i B(\lambda, \mu)) \Rightarrow$

$$\Theta_{A,\lambda}\left(-\frac{1}{\tau}\right) = \frac{(-i\tau)^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{\mu \in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r} \sum_{n \in \mu + \mathbb{Z}^r} \exp(2\pi i Q(n)\tau + 2\pi i B(\lambda, \mu))$$

$$= \frac{(-i\tau)^{\frac{r}{2}}}{\sqrt{\det(A)}} \sum_{\mu \in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r} \exp(2\pi i B(\lambda, \mu))\Theta_{A,\mu}(\tau) \qquad \square$$

Bemerkung

Bemerkung (Erweiterung auf Γ_1)

$r \in \mathbb{N}$ gerade; $A \in M_{r,r}(\mathbb{Z})$ gerade, positiv definit; $\lambda \in A^{-1}\mathbb{Z}^r \Rightarrow$

$$\forall \gamma \in \Gamma_1 : \Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma) \in \mathrm{span}_{\mathbb{C}}\left(V := \left\{\Theta_{A,\mu} \,|\, \mu \in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r\right\}\right)$$

Beweis.

$\Theta_{A,\lambda}(T.\tau) = \Theta_{A,\lambda}(\tau + 1) \in \mathrm{span}_{\mathbb{C}}(V)\wedge$
$\Theta_{A,\lambda}(S.\tau) = \Theta_{A,\lambda}\left(-\frac{1}{\tau}\right) \in \mathrm{span}_{\mathbb{C}}(V)\wedge\ \Gamma_1 = \langle T, S\rangle\wedge$
$|$ linkslinear, $\forall \gamma_1, \gamma_2 \in \Gamma_1 : (\Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma_1))|_{\frac{r}{2}}(\gamma_2) = \Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma_1\gamma_2) \Rightarrow$

$$(\Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma))(\tau) = \frac{1}{(c\tau + d)^{\frac{r}{2}}}\Theta_{A,\lambda}(\gamma.\tau) \in \mathrm{span}_{\mathbb{C}}(V)$$

□

Hilfsdefinition

Definition

$A \in M_{r,r}(\mathbb{Z})$ gerade; $\lambda \in A^{-1}\mathbb{Z}^r$; $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma_1$; $c \neq 0 \rightarrow$

$$S_\gamma(\lambda) := \sum_{\mu \in \mathbb{Z}^r / c\mathbb{Z}^r} \exp\left(2\pi i \frac{a}{c} Q(\mu + \lambda)\right)$$

Bemerkung

$$\forall \mu \in \mathbb{Z}^r / c\mathbb{Z}^r : \exp\left(2\pi i \frac{a}{c} Q(\mu + c\mathbb{Z}^r)\right) = \exp\left(2\pi i \frac{a}{c} Q(\mu)\right)$$

$\Rightarrow S_\gamma(\lambda)$ wohldefiniert (unabhängig von Restklassenwahl)$\wedge$
$\forall \lambda \in A^{-1}\mathbb{Z}^r / \mathbb{Z}^r : S_\gamma(\lambda + \mathbb{Z}^r) = S_\gamma(\lambda)$

Achtung

Strichformel

Theorem (Strichformel)

*$r \in \mathbb{N}$ gerade; $A \in M_{r,r}(\mathbb{Z})$ gerade, positiv definit; $\lambda \in A^{-1}\mathbb{Z}^r$;
$\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma_1$; $c \neq 0 \Rightarrow$*

$$\Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma) = \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{2}}\sqrt{\det(A)}} \cdot \sum_{\lambda' \in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r} \exp(-2\pi i[bB(\lambda,\lambda') + bdQ(\lambda')]) \cdot S_\gamma(\lambda + d\lambda') \cdot \Theta_{A,\lambda'}$$

1. $S_{-\gamma} = S_\gamma \wedge |_{\frac{r}{2}}(-\gamma) = (-1)^{\frac{r}{2}}|_{\frac{r}{2}}(\gamma) \Rightarrow \gamma- > -\gamma \Rightarrow$ Œ $\quad c > 0$
2. $V_c := \operatorname{diag}(c,1) \Rightarrow \gamma = \left(\begin{smallmatrix} c^{-1} & 0 \\ 0 & 1 \end{smallmatrix}\right)\left(\begin{smallmatrix} a & bc \\ 1 & d \end{smallmatrix}\right)\left(\begin{smallmatrix} c & 0 \\ 0 & 1 \end{smallmatrix}\right) = V_c^{-1} T^a S T^d V_c$
3. $1 \Rightarrow$

$$\begin{aligned}
(\Theta_{A,\lambda}|_{\frac{r}{2}}(V_c^{-1}))(\tau) &= \frac{(\det(V_c^{-1}))^{\frac{r}{4}}}{1^k}\Theta_{A,\lambda}\left(\frac{\tau}{c}\right) \\
&= c^{-\frac{r}{4}} \sum_{n\in\lambda+\mathbb{Z}^r} q^{\frac{Q(n)}{c}} \\
&= c^{-\frac{r}{4}} \sum_{m\in\frac{\lambda}{c}+\frac{1}{c}\mathbb{Z}^r} q^{cQ(m)} \\
&= c^{-\frac{r}{4}} \sum_{\mu\in\mathbb{Z}_c^r} \sum_{m\in\frac{\lambda+\mu}{c}+\mathbb{Z}^r} q^{cQ(m)} \\
&= c^{-\frac{r}{4}} \sum_{\mu\in\mathbb{Z}_c^r} \left(\Theta_{cA,\frac{\lambda+\mu}{c}}\right)(\tau)
\end{aligned}$$

4. $\star(A) \wedge 1 \Rightarrow cA$ positiv definit, gerade $\wedge$
$\forall \mu \in \mathbb{Z}^r : \forall \lambda \in A^{-1}\mathbb{Z}^r : cA\frac{\lambda+\mu}{c} = A\lambda + A\mu \in \mathbb{Z}^r \to \frac{\lambda+\mu}{c} \in (cA)^{-1}\mathbb{Z}^r$
$\Rightarrow$ Transformationsformeln auf $\Theta_{cA,\frac{\lambda+\mu}{c}}$ anwendbar $\Rightarrow$

$$\Theta_{cA,\frac{\lambda+\mu}{c}}|_{\frac{r}{2}}(T^a) \overset{T(i)}{=} \exp\left(2\pi i a c Q\left(\frac{\lambda+\mu}{c}\right)\right)\Theta_{cA,\frac{\lambda+\mu}{c}}$$
$$= \exp\left(2\pi i \frac{a}{c} Q(\lambda+\mu)\right)\Theta_{cA,\frac{\lambda+\mu}{c}}$$

5.

$$\Theta_{cA,\frac{\lambda+\mu}{c}}|_{\frac{r}{2}}(T^a S) \overset{4}{=} \exp\left(2\pi i \frac{a}{c} Q(\lambda+\mu)\right)\Theta_{cA,\frac{\lambda+\mu}{c}}|_{\frac{r}{2}}(S) \overset{T(ii)}{=}$$
$$\exp(\ldots)\frac{(-i)^{\frac{r}{2}}}{\sqrt{\det(cA)}} \sum_{\nu\in(cA)^{-1}\mathbb{Z}^r/\mathbb{Z}^r} \exp(2\pi i B(\lambda+\mu,\nu))\Theta_{cA,\nu}$$

6.

$$\Theta_{cA,\frac{\lambda+\mu}{c}}\big|_{\frac{r}{2}}\left(T^aST^d\right) \overset{5}{=}$$

$$\exp(\dots)\frac{(-i)^{\frac{r}{2}}}{\sqrt{\det(cA)}}\sum_{\nu\in(cA)^{-1}\mathbb{Z}^r/\mathbb{Z}^r}\exp(2\pi iB(\lambda+\mu,\nu))\Theta_{cA,\nu}|_{\frac{r}{2}}(T^d) \overset{T(i)}{=}$$

$$\exp(\dots)\frac{(-i)^{\frac{r}{2}}}{\sqrt{\det(cA)}}\sum_{\nu\in(cA)^{-1}\mathbb{Z}^r/\mathbb{Z}^r}\exp(2\pi i[B(\lambda+\mu,\nu)+cdQ(\nu)])\Theta_{cA,\nu} =$$

$$\frac{(-i)^{\frac{r}{2}}}{\sqrt{\det(cA)}}\sum_{\nu\in(cA)^{-1}\mathbb{Z}^r/\mathbb{Z}^r}\exp\left(2\pi i\left[\frac{a}{c}Q(\lambda+\mu)+B(\lambda+\mu,\nu)+cdQ(\nu)\right]\right)\Theta_{cA,\nu}$$

7.

$$\Theta_{cA,\lambda}|_{\frac{r}{2}}\left(V_c^{-1}T^aST^d\right) \overset{3}{=} \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{4}}\sqrt{\det(cA)}} \cdot \sum_{\substack{\mu\in\mathbb{Z}_c^r \\ \nu\in(cA)^{-1}\mathbb{Z}^r/\mathbb{Z}^r}} \exp\left(2\pi i\left[\frac{a}{c}Q(\lambda+\mu)+B(\lambda+\mu,\nu)+cdQ(\nu)\right]\right)\Theta_{cA,\nu}$$

8. $7\wedge\forall\nu\in(cA)^{-1}\mathbb{Z}^r/\mathbb{Z}^r:\exists\lambda'\in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r:\exists\mu'\in\mathbb{Z}_c^r:\nu=\frac{\lambda'+\mu'}{c}\Rightarrow$

$$\Theta_{cA,\lambda}|_{\frac{r}{2}}\left(V_c^{-1}T^aST^d\right) = \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{4}}\sqrt{\det(cA)}} \sum_{\substack{\mu,\mu'\in\mathbb{Z}_c^r \\ \lambda'\in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r}} \exp\left(2\pi i\left[\frac{a}{c}Q(\lambda+\mu)+\frac{1}{c}B(\lambda+\mu,\lambda'+\mu')+\frac{d}{c}Q(\lambda'+\mu')\right]\right)\Theta_{cA,\frac{\lambda'+\mu'}{c}}$$

9. $x := \lambda + \mu, y := \lambda' + \mu' \wedge$
$\frac{a}{c}Q(x)+\frac{1}{c}B(x,y)+\frac{d}{c}Q(y) = \frac{a}{c}Q(x+dy)-bB(x,y)-bdQ(y)\wedge$
$B(\lambda+\mu,\lambda'+\mu')-B(\lambda,\lambda')=B(\lambda,\lambda')+B(\lambda',\mu)+B(\mu,\mu')\in\mathbb{Z}\wedge$
$Q(\lambda'+\mu')-Q(\lambda')=B(\lambda',\mu')+Q(\mu')\in\mathbb{Z}\Rightarrow$

$$\exp\left(2\pi i\left[\frac{a}{c}Q(\lambda+\mu)+\frac{1}{c}B(\lambda+\mu,\lambda'+\mu')+\frac{d}{c}Q(\lambda'+\mu')\right]\right)=$$
$$\exp\left(2\pi i\left[\frac{a}{c}Q(\lambda+\mu+d\lambda'+d\mu')-bB(\lambda,\lambda')-bdQ(\lambda')\right]\right)$$

10. $8\wedge 9\wedge \mu\mapsto\mu-d\mu'\Rightarrow$

$$\Theta_{cA,\lambda}|_{\frac{r}{2}}\left(V_c^{-1}T^aST^d\right)=\frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{4}}\sqrt{\det(cA)}}\cdot\sum_{\substack{\mu,\mu'\in\mathbb{Z}_c^r\\ \lambda'\in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r}}$$
$$\exp\left(2\pi i\left[\frac{a}{c}Q(\lambda+\mu+d\lambda')-bB(\lambda,\lambda')-bdQ(\lambda')\right]\right)\Theta_{cA,\frac{\lambda'+\mu'}{c}}$$

11.

$$\Theta_{cA,\lambda}|_{\frac{r}{2}}\left(V_c^{-1}T^aST^d\right)$$

$$\overset{10}{=} \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{4}}\sqrt{\det(cA)}}\frac{c^{-\frac{r}{4}}}{c^{-\frac{r}{4}}}\sum_{\substack{\mu\in\mathbb{Z}_c^r\\ \lambda'\in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r}}$$

$$\exp\left(2\pi i\left[\frac{a}{c}Q(\lambda+\mu+d\lambda')-bB(\lambda,\lambda')-bdQ(\lambda')\right]\right)\sum_{\mu'\in\mathbb{Z}_c^r}\Theta_{cA,\frac{\lambda'+\mu'}{c}}$$

$$\overset{3}{=} \frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{2}}\sqrt{\det(A)}}\sum_{\substack{\mu\in\mathbb{Z}_c^r\\ \lambda'\in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r}}$$

$$\exp\left(2\pi i\left[\frac{a}{c}Q(\lambda+\mu+d\lambda')-bB(\lambda,\lambda')-bdQ(\lambda')\right]\right)\Theta_{A,\lambda'}|_{\frac{r}{2}}\left(V_c^{-1}\right)$$

12.

$$\Theta_{cA,\lambda}|_{\frac{r}{2}}\left(V_c^{-1}T^aST^d\right)$$

$$\stackrel{11}{=}\frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{2}}\sqrt{\det(A)}}\sum_{\lambda'\in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r}\exp(-2\pi i[bB(\lambda,\lambda')+bdQ(\lambda')])\cdot$$

$$\left[\sum_{\mu\in\mathbb{Z}_c^r}\exp\left(\frac{a}{c}Q(\mu+\lambda+d\lambda')\right)\right]\Theta_{A,\lambda'}|_{\frac{r}{2}}\left(V_c^{-1}\right)$$

$$=\frac{(-i)^{\frac{r}{2}}}{c^{\frac{r}{2}}\sqrt{\det(A)}}\sum_{\lambda'\in A^{-1}\mathbb{Z}^r/\mathbb{Z}^r}\exp(-2\pi i[bB(\lambda,\lambda')+bdQ(\lambda')])\cdot$$

$$S_\gamma(\lambda+d\lambda')\Theta_{A,\lambda'}|_{\frac{r}{2}}\left(V_c^{-1}\right)$$

13. $2,11\Rightarrow\Theta_{A,\lambda}|_{\frac{r}{2}}(\gamma)=\Theta_{cA,\frac{\lambda+\mu}{c}}|_{\frac{r}{2}}\left(V_c^{-1}T^aST^dV_c\right)$ □

Ende

Vielen Dank für Ihre Aufmerksamkeit!