

Anonymity Networks

Laslo Hunhold

Mathematisches Institut
Universität zu Köln

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In the lecture '*Information Theory and Statistical Physics*' by Prof. Dr. Johannes BERG

motivation

motivation

- ▶ hide initiator of a message in a computer network

motivation

- ▶ hide initiator of a message in a computer network
- ▶ safe whistleblowing under corporate and state surveillance

motivation

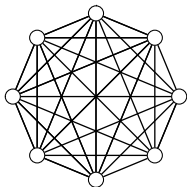
- ▶ hide initiator of a message in a computer network
- ▶ safe whistleblowing under corporate and state surveillance
- ▶ 'deniable communication'

motivation

- ▶ hide initiator of a message in a computer network
- ▶ safe whistleblowing under corporate and state surveillance
- ▶ 'deniable communication'
- ▶ decentralized

idea

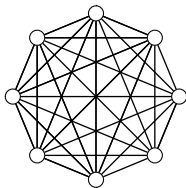
idea



node network participant

link possible message path

idea

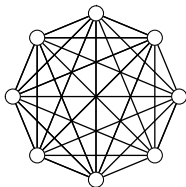


node network participant

link possible message path

- ▶ all nodes have equal weight

idea

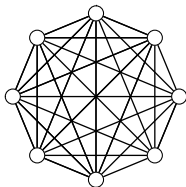


node network participant

link possible message path

- ▶ all nodes have equal weight
- ▶ message unmodifiable, only receiver is known

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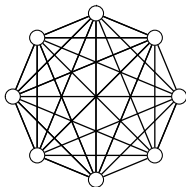


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- ▶ each node on path: biased coin flip: forward or deliver

idea

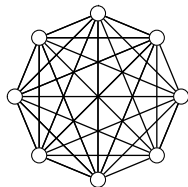


node network participant

link possible message path

- ▶ all nodes have equal weight
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- ▶ each node on path: initiator or just forwarder?

idea



node network participant

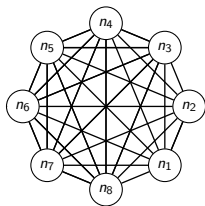
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→ message initiator gets lost in the crowd

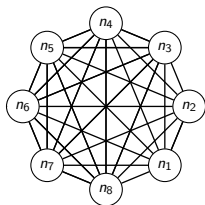
model

model



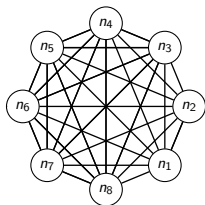
- ▶ N nodes n_1, \dots, n_N with $\mathbb{P}(n_i \text{ is initiator}) =: \mathbb{P}(X = n_i) =: p_i$

model



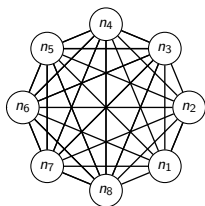
- ▶ N nodes n_1, \dots, n_N with $\mathbb{P}(n_i \text{ is initiator}) =: \mathbb{P}(X = n_i) =: p_i$
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- ▶ forwarding probability λ

if message received **then**

flip biased coin $\mathbb{P}(\text{heads}) = \lambda$

if heads **then**

forward to a uniformly chosen node

else

deliver to receiver

end if

end if

degree of anonymity

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best case $\bar{X} := X : \forall i \in \{1, \dots, N\} : p_i = \frac{1}{N}$

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worst case $\underline{X} := X : \forall i \in \{1, \dots, N\} \setminus \{j\} : p_i = 0 \wedge p_j = 1$

degree of anonymity

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degree of anonymity

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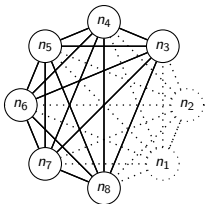
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$$d(X) := 1 - \frac{\bar{H} - H(X)}{\bar{H}} = \frac{H(X)}{\bar{H}} \in [0, 1]$$

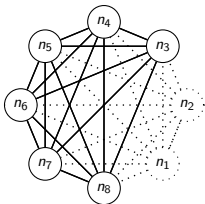
corruption

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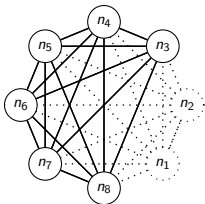
- ▶ $0 \leq C < N$ corrupt nodes (incoming message passer known)

corruption



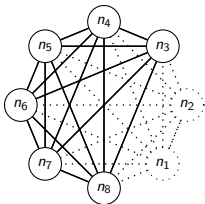
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corruption



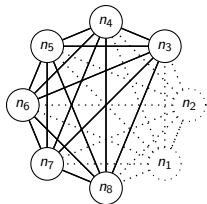
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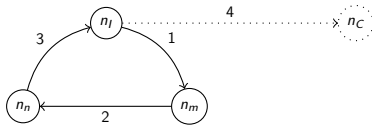
$\mathbb{P}(\text{passer is initiator}) > \frac{1}{2} \rightarrow \text{unmasked}$

analysis

events

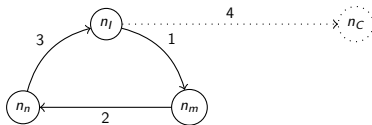
analysis

events



analysis

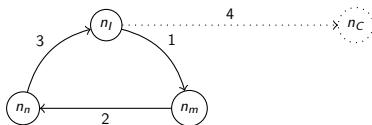
events



let $k > 0$

analysis

events

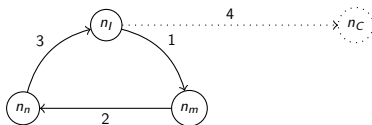


let $k > 0$

$H_k :=$ first corrupt node is at the k th path-position

analysis

events



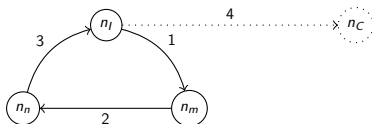
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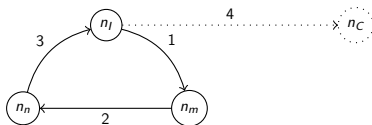
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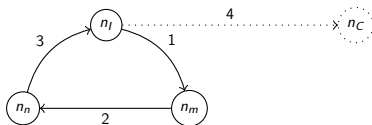
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$$\mathbb{P}(\text{passer is initiator}) = \mathbb{P}(I|H_{1+})$$

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note: $H_1 \rightarrow I$, but $I \not\rightarrow H_1$

analysis

general probability I

$$\mathbb{P}(I|H_{1+}) = \frac{N - \lambda(N - C - 1)}{N}$$

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$$\mathbb{P}(I|H_{2+}) = \frac{1}{N - C}$$

analysis

general probability II

$$\mathbb{P}(I) \stackrel{TP}{=} \mathbb{P}(H_1) \mathbb{P}(I|H_1) + \mathbb{P}(H_{2+}) \mathbb{P}(I|H_{2+})$$

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□

good node $\mathbb{P}(\text{good node } i \text{ is initiator}) = \frac{1 - \mathbb{P}(I|H_{1+})}{N - C - 1} = \frac{\lambda}{N} < \frac{1}{N} \leq \frac{1}{2}$

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 \Rightarrow all good nodes besides passer are innocent

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corrupt node $\mathbb{P}(\text{corrupt node } i \text{ is initiator}) = 0$

analysis

passer innocence

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proof:

$$\frac{1}{2} \geq \mathbb{P}(I|H_{1+})$$

analysis

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$$\frac{1}{2} \geq \mathbb{P}(I|H_{1+}) = \frac{N - \lambda(N - C - 1)}{N}$$

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$$\frac{1}{2} \geq \mathbb{P}(I|H_{1+}) = \frac{N - \lambda(N - C - 1)}{N} \quad \Bigg| \quad \left(\lambda - \frac{1}{2}\right) > 0$$

$$\Leftrightarrow N \geq \frac{1}{1 - \frac{1}{2 \cdot \lambda}} \cdot (C + 1)$$



analysis

degree of anonymity

analysis

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$$d(X) = - \frac{C \cdot 0 + \mathbb{P}(I|H_{1+}) \cdot \ln(\mathbb{P}(I|H_{1+})) + (N - C - 1) \cdot \frac{\lambda}{N} \cdot \ln\left(\frac{\lambda}{N}\right)}{\ln(N - C)} =$$

analysis

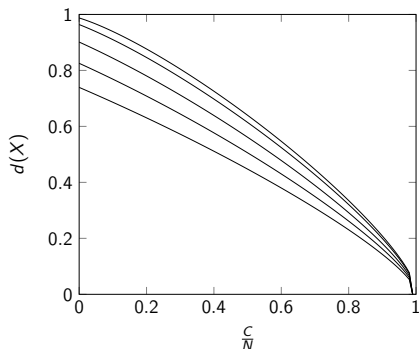
degree of anonymity

$$\begin{aligned}d(X) &= - \frac{C \cdot 0 + \mathbb{P}(I|H_{1+}) \cdot \ln(\mathbb{P}(I|H_{1+})) + (N - C - 1) \cdot \frac{\lambda}{N} \cdot \ln\left(\frac{\lambda}{N}\right)}{\ln(N - C)} = \\ &= \dots = \frac{(N - \lambda \cdot (N - C - 1)) \cdot \ln\left(\frac{N}{N - \lambda \cdot (N - C - 1)}\right) + \lambda \cdot (N - C - 1) \cdot \ln\left(\frac{N}{\lambda}\right)}{N \cdot \ln(N - C)}\end{aligned}$$

analysis

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conclusion

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- ▶ full paper on <http://frign.de/>