

Modern Methods for Signal Analysis: Empirical Mode Decomposition Theory and Hybrid Operator-Based Methods Using B-Splines

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Mathematisches Institut
Universität zu Köln

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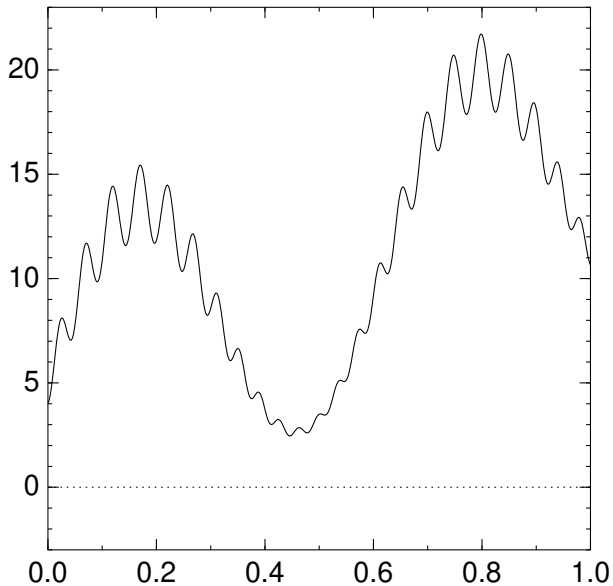
Prof. Dr. Angela KUNOTH

29th October 2018



introduction

example: multicomponent input signal s



introduction

components?

introduction

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definition: intrinsic mode function (IMF)

$$\mathcal{I} \ni: u(t) = a(t) \cdot \cos(\phi(t))$$

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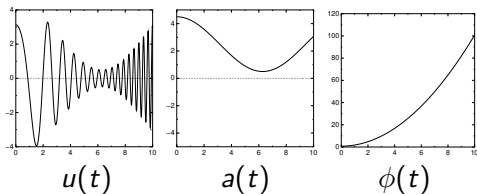
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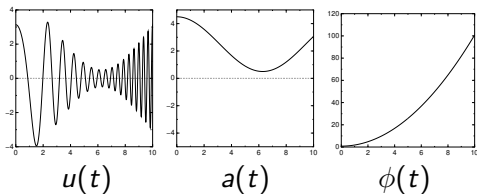
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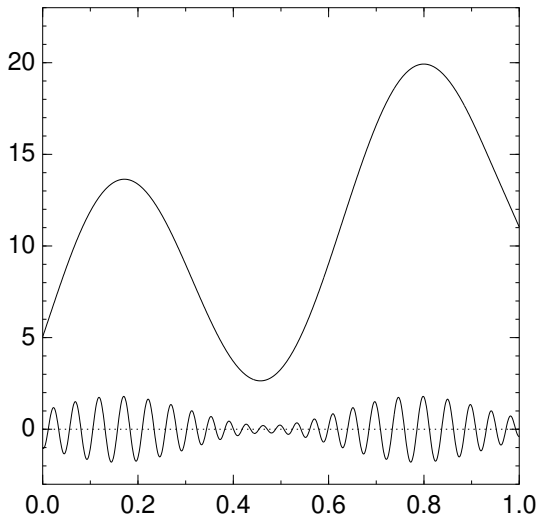


definition: multicomponent signal

$$s := \sum_{k=0}^w u_k(t) + r_{w+1}(t), \quad u_k \in \mathcal{I}$$

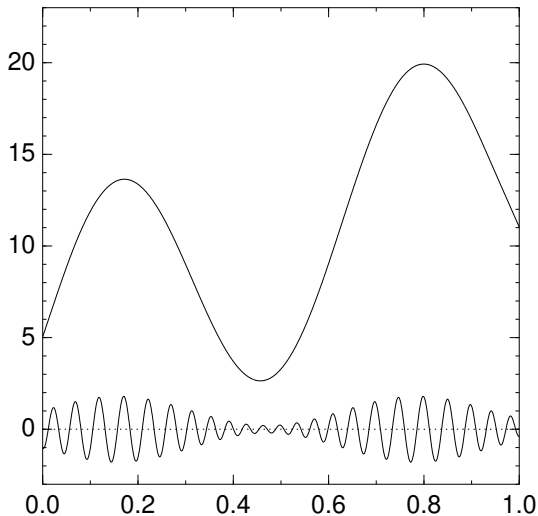
introduction

example: decomposition of s into residual r_1 (upper) and IMF u_0 (lower)



introduction

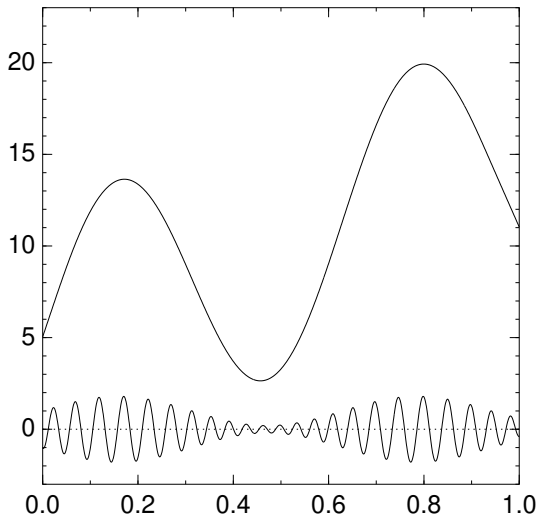
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how?

introduction

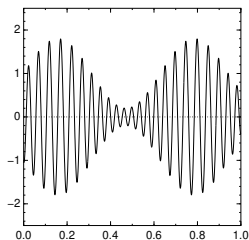
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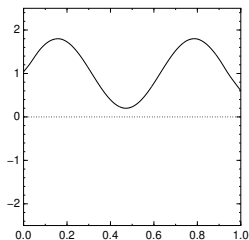
how? later

introduction

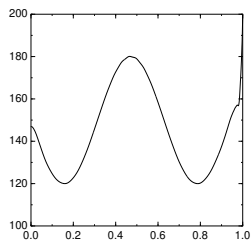
example: extraction of a_0 and ϕ'_0 from u_0



$$u_0(t) = a_0(t) \cdot \cos(\phi_0(t))$$



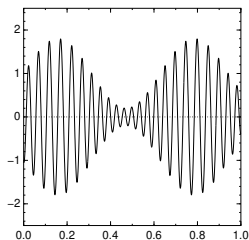
$$a_0(t)$$



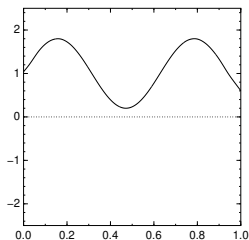
$$\phi'_0(t)$$

introduction

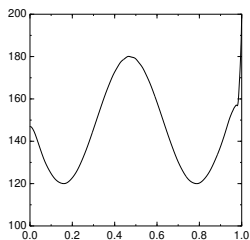
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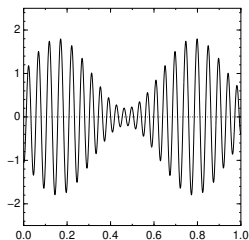


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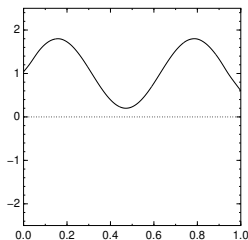
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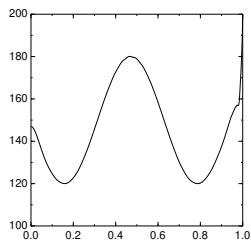
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$$u_0(t) = a_0(t) \cdot \cos(\phi_0(t))$$



$$a_0(t)$$



$$\phi'_0(t)$$

how? using differential operators!

operator-based empirical mode decomposition

idea

operator-based empirical mode decomposition

idea

find differential operator $\mathcal{D}_{(a,\phi)}$ such that for

$$u = a(t) \cdot \cos(\phi(t)) \in \mathcal{I}$$

it holds

$$\mathcal{D}_{(a,\phi)} u = 0.$$

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find differential operator $\mathcal{D}_{(a,\phi)}$ such that for

$$u = a(t) \cdot \cos(\phi(t)) \in \mathcal{I}$$

it holds

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extract (a_k, ϕ_k) from u_k via

$$(a_k, \phi_k) = \arg \min_{(a,\phi)} \left\| \mathcal{D}_{(a,\phi)} u_k \right\|.$$

operator-based empirical mode decomposition

example differential-operator (Guo et al., 2017)

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general case (a, ϕ) does not yield convex optimization problem.

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simple case $(1, \phi)$ considered, i.e. $u(t) = \cos(\phi(t))$

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$$\mathcal{D}_{(1, \phi)}^G f := \left(\frac{1}{(\phi')^2} \right) \cdot f'' + \frac{1}{2} \cdot \left(\frac{1}{(\phi')^2} \right)' \cdot f' + f$$

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linear operator in B-Spline-expression for

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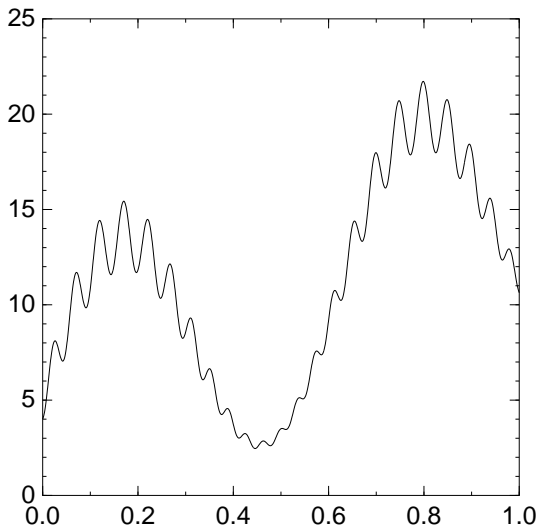
problem how to extract the amplitude from the IMF?

IMF extraction

traditional method (1/4)

IMF extraction

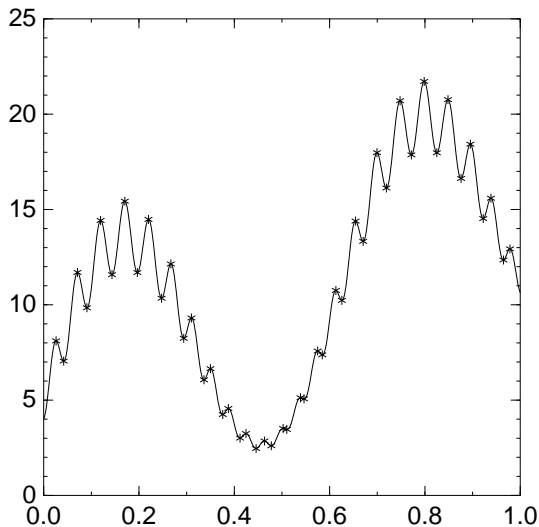
traditional method (1/4)



begin with input signal s

IMF extraction

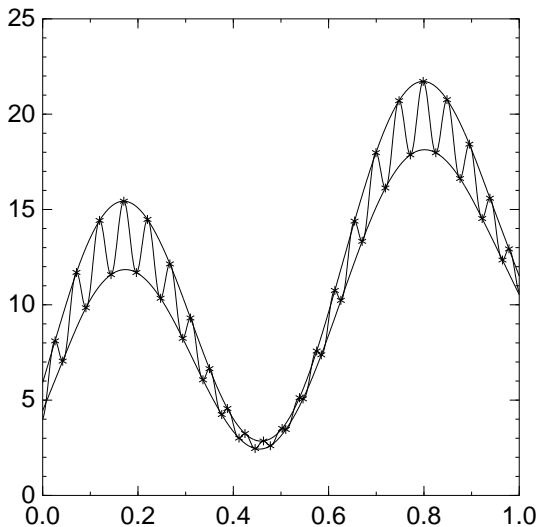
traditional method (2/4)



identify local maxima and minima

IMF extraction

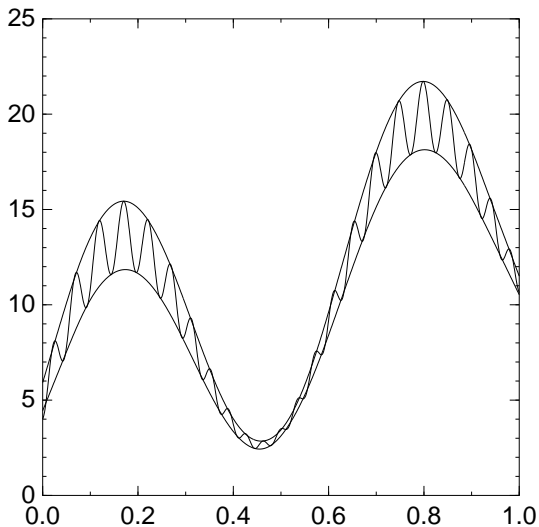
traditional method (3/4)



interpolate local maxima and minima

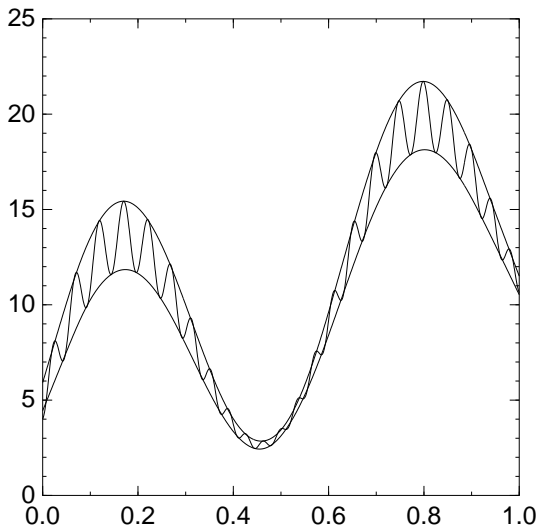
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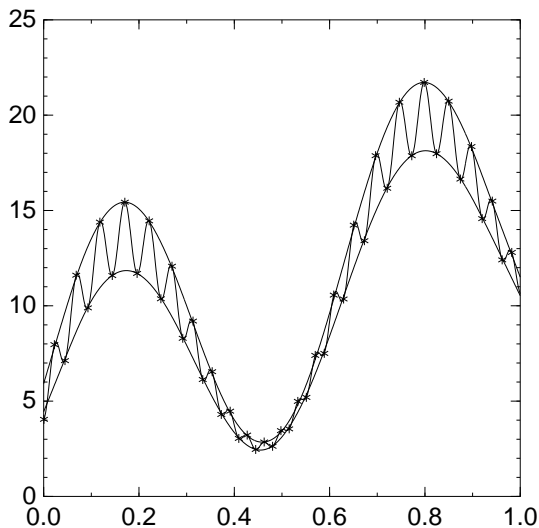
problem envelope cuts s , see (Huang, Kunoht, 2012)

IMF extraction

proposed method (1/4) (first iteration)

IMF extraction

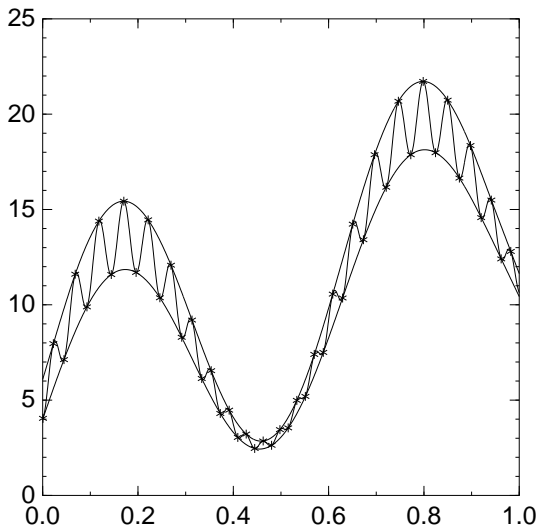
proposed method (1/4) (first iteration)



identify matching gradient points of s , filter with curvature

IMF extraction

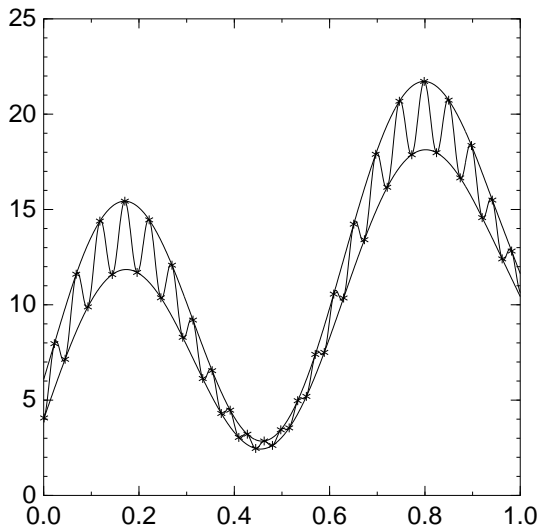
proposed method (2/4) (first iteration)



interpolate matching gradient points

IMF extraction

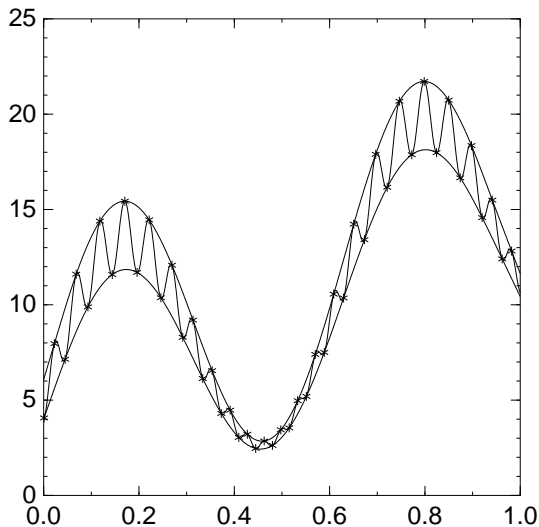
proposed method (3/4) (second iteration)



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proposed method (4/4) (second iteration)



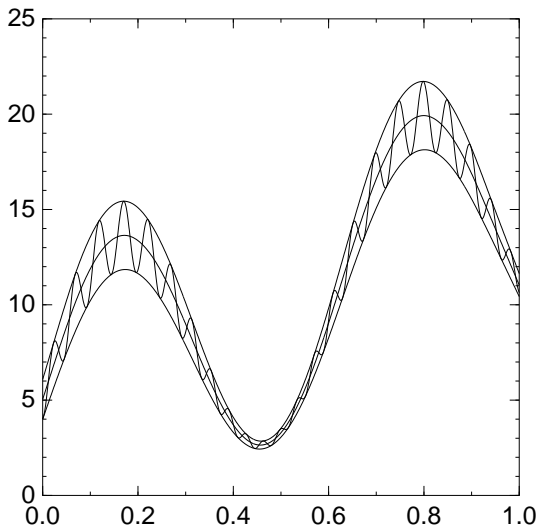
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IMF extraction

extracting the residual

IMF extraction

extracting the residual



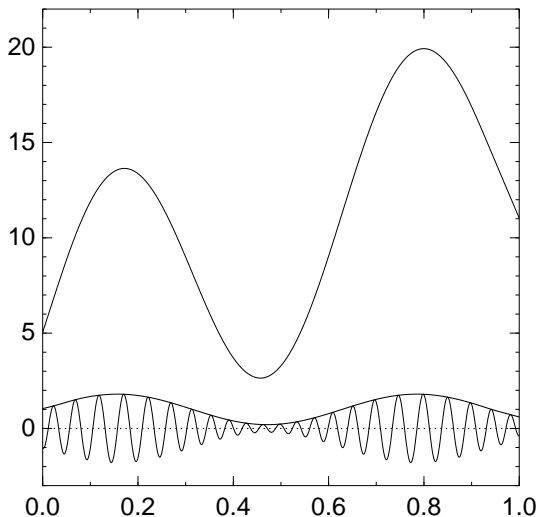
calculate residual r_1 as mean of upper and lower envelope

IMF extraction

final signal separation

IMF extraction

final signal separation



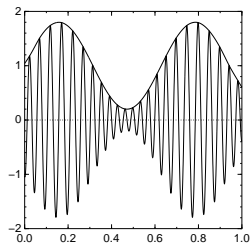
calculate IMF u_0 and a_0 by subtracting r_1 from s_1 and u. envelope

IMF extraction

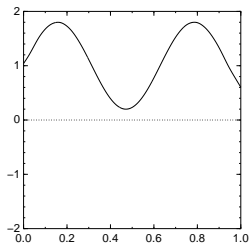
final IMF separation (1/2)

IMF extraction

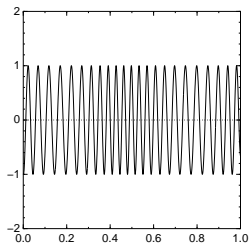
final IMF separation (1/2)



$u_0(t), a_0(t)$



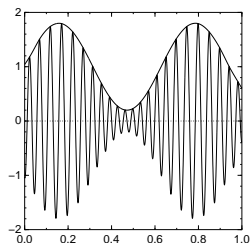
$a_0(t)$



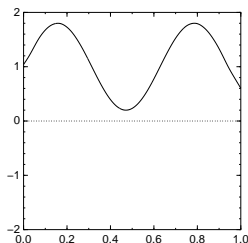
$u_0(t)/a_0(t)$

IMF extraction

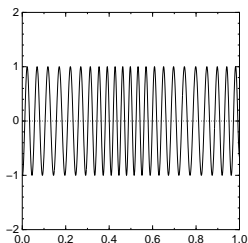
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$u_0(t), a_0(t)$



$a_0(t)$



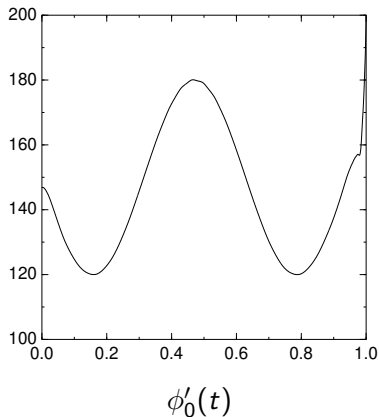
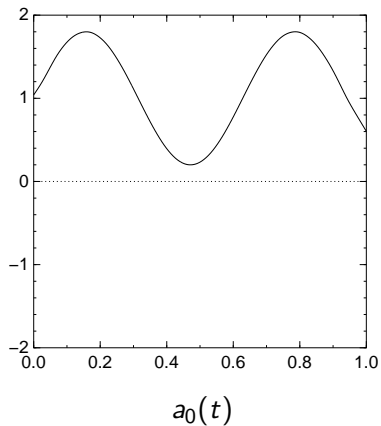
$u_0(t)/a_0(t)$

last step apply simple operator-based method to

$$u_0(t)/a_0(t) = \cos(\phi(t))$$

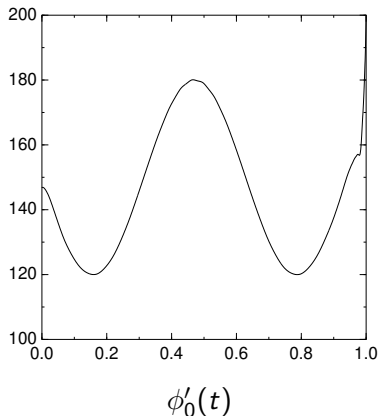
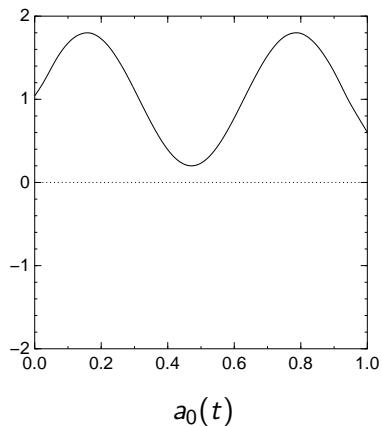
IMF extraction

final IMF separation (2/2)



IMF extraction

final IMF separation (2/2)



designation: hybrid operator-based methods

theoretical results

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optimization problem for input signal s

$$\begin{aligned} \min_{(a, \phi)} \quad & \|a \cdot \cos(\phi) - s\| \\ \text{s.t.} \quad & a \cdot \cos(\phi) \text{ IMF.} \end{aligned}$$

theoretical results

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justification for heuristics of the form

$$\min_{(a,\phi)} \quad \|\text{cost}\| + \|\text{regularization operator}\|$$

numerical results

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- ▶ C-toolbox

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 - ▶ B-Spline precomputation

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 - ▶ sparse linear (weighted) least-squares fits

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 - ▶ complexity
- ▶ proposed envelope procedure converges

outlook

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- ▶ substitute GSL for a more specific library

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- ▶ find higher-order linear operators (or prove non-existence), as

$$\mathcal{D}_{(1,\phi)}^G f := \left(\frac{1}{(\phi')^2} \right) \cdot f'' + \frac{1}{2} \cdot \left(\frac{1}{(\phi')^2} \right)' \cdot f' + f$$

only goes up to f''