Modern Methods for Signal Analysis: Empirical Mode Decomposition Theory and Hybrid Operator-Based Methods Using B-Splines

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hosted by

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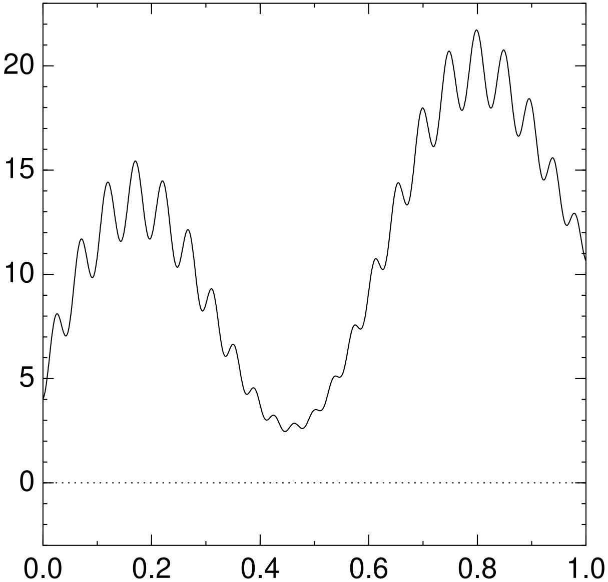
introduction

example: multicomponent input signal s



introduction

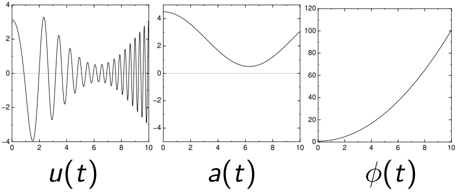
components?

definition: intrinsic mode function (IMF)

$$\mathcal{I} \ni: u(t) = a(t) \cdot \cos(\phi(t))$$

is an IMF with accuracy $\varepsilon \in (0,1)$ if and only if for all $t \in \mathbb{R}$

- $a(t) \geq 0$, $\phi'(t) > 0$ (physical reasonability),
- $|a'(t)| \leq \varepsilon|\phi'(t)|$ (slowly varying amplitude),
- $|\phi''(t)| \leq \varepsilon|\phi'(t)|$ (slowly varying frequency).

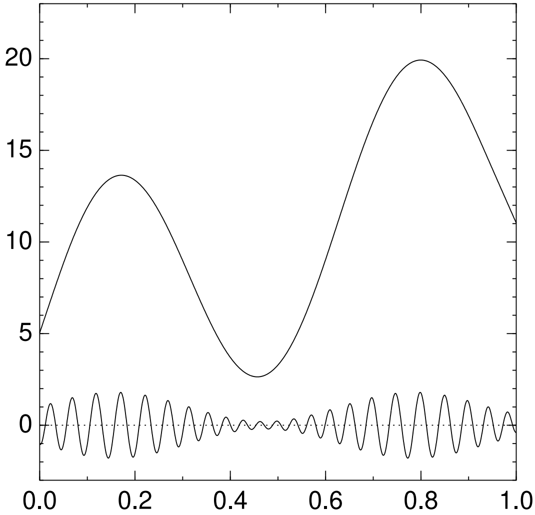


definition: multicomponent signal

$$s := \sum_{k=0}^{w} u_k(t) + r_{w+1}(t), \quad u_k \in \mathcal{I}$$

introduction

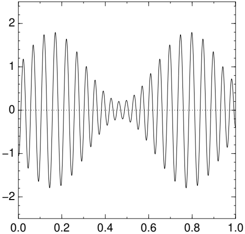
example: decomposition of s into residual r_1 (upper) and IMF u_0 (lower)



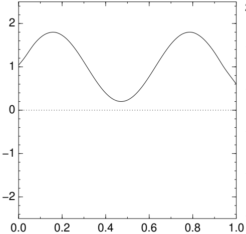
how? later

introduction

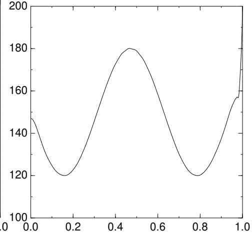
example: extraction of a_0 and ϕ_0' from u_0



$u_0(t) = a_0(t) \cdot \cos(\phi_0(t))$



$a_0(t)$



$\phi_0'(t)$

how? using differential operators!

operator-based empirical mode decomposition

idea

find differential operator $\mathcal{D}_{(a,\phi)}$ such that for

$$u = a(t) \cdot \cos(\phi(t)) \in \mathcal{I}$$

it holds

$$\mathcal{D}_{(a,\phi)} u = 0.$$

extract (a_k, ϕ_k) from u_k via

$$(a_k, \phi_k) = \arg\min_{(a,\phi)} \left\| \mathcal{D}_{(a,\phi)} u_k \right\|.$$

operator-based empirical mode decomposition

example differential-operator (Guo et al., 2017)

general case (a, ϕ) does not yield convex optimization problem.

simple case $(1, \phi)$ considered, i.e. $u(t) = \cos(\phi(t))$

$$\mathcal{D}^G_{(1,\phi)} f := \left(\frac{1}{(\phi')^2} \right) \cdot f'' + \frac{1}{2} \cdot \left(\frac{1}{(\phi')^2} \right)' \cdot f' + f$$

$$\mathcal{D}^G_{(1,\phi)} u = 0$$

linear operator in B-Spline-expression for

$$\Omega[\phi] := \frac{1}{(\phi')^2}$$

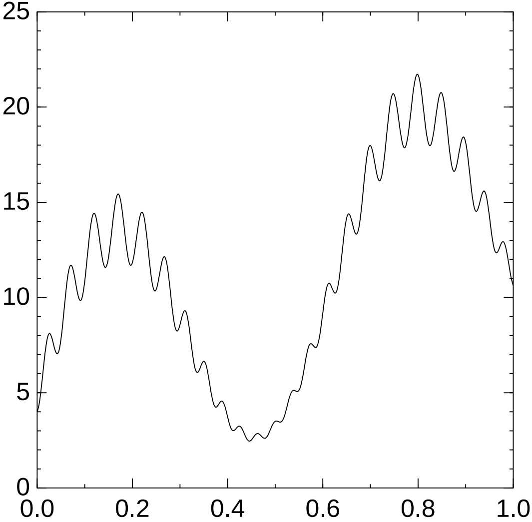
$\rightarrow$ convex optimization problem yielding Ω,

$$\phi' = \frac{1}{\sqrt{\Omega}}$$

problem how to extract the amplitude from the IMF?

IMF extraction

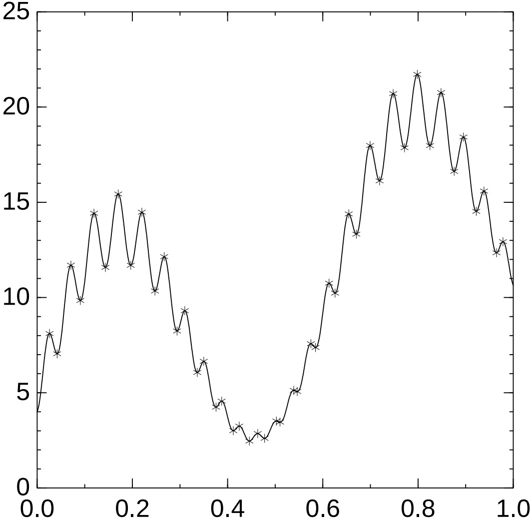
traditional method (1/4)



begin with input signal s

IMF extraction

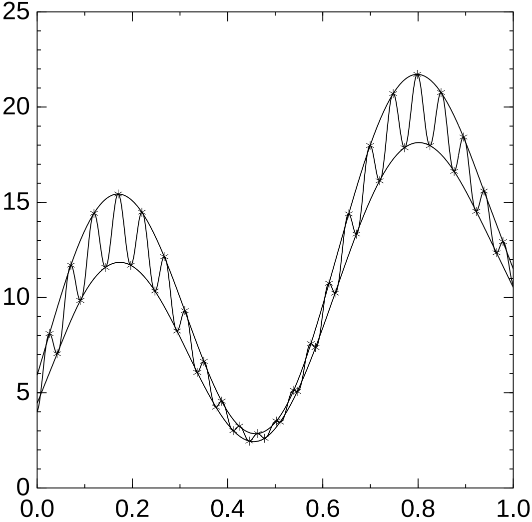
traditional method (2/4)



identify local maxima and minima

IMF extraction

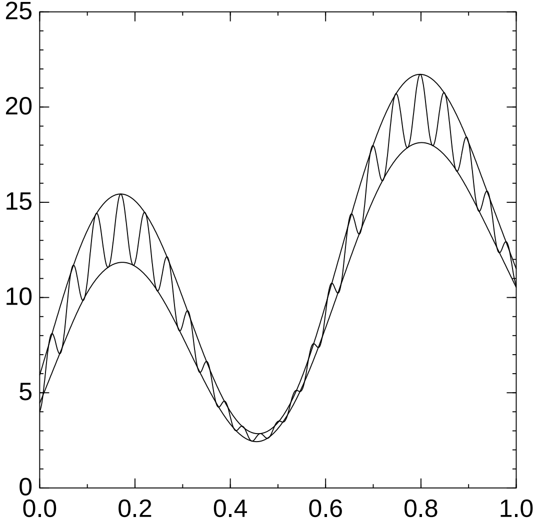
traditional method (3/4)



interpolate local maxima and minima

IMF extraction

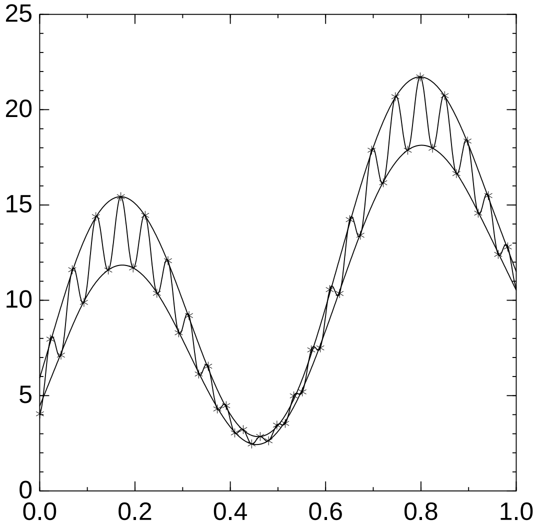
traditional method (3/4)



problem envelope cuts s, see (Huang, Kunoth, 2012)

IMF extraction

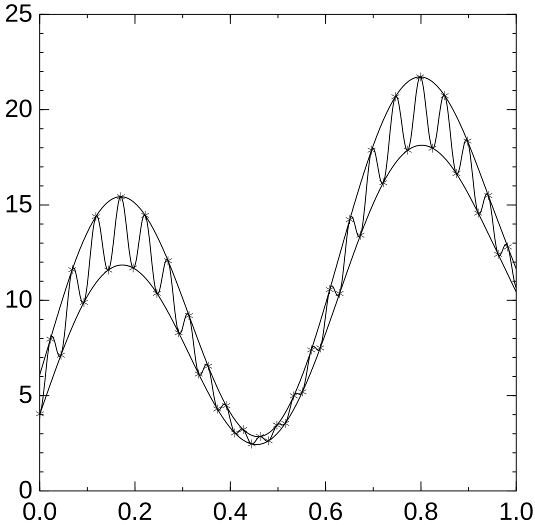
proposed method (1/4) (first iteration)



identify matching gradient points of s, filter with curvature

IMF extraction

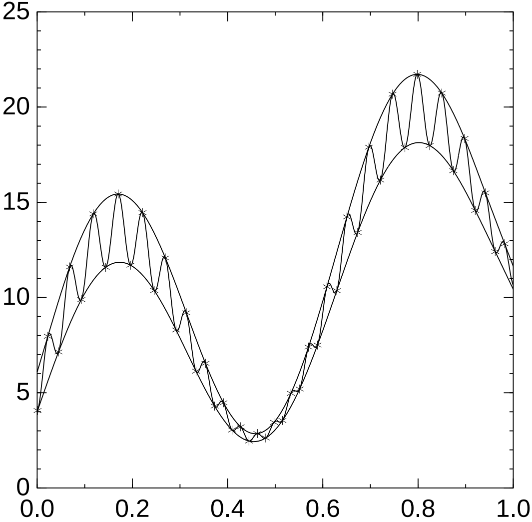
proposed method (2/4) (first iteration)



interpolate matching gradient points

IMF extraction

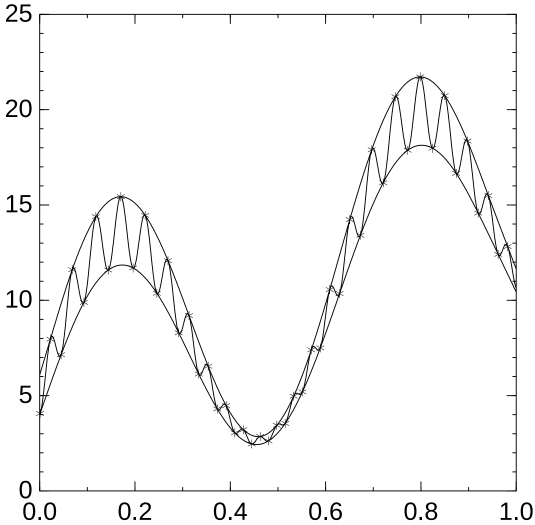
proposed method (3/4) (second iteration)



identify matching gradient points of s, filter with curvature

IMF extraction

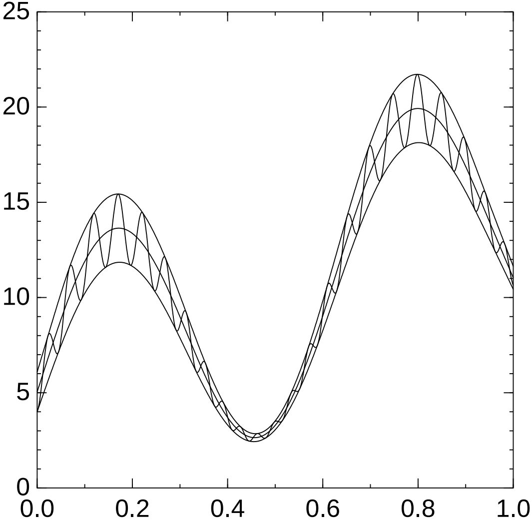
proposed method (4/4) (second iteration)



interpolate matching gradient points

IMF extraction

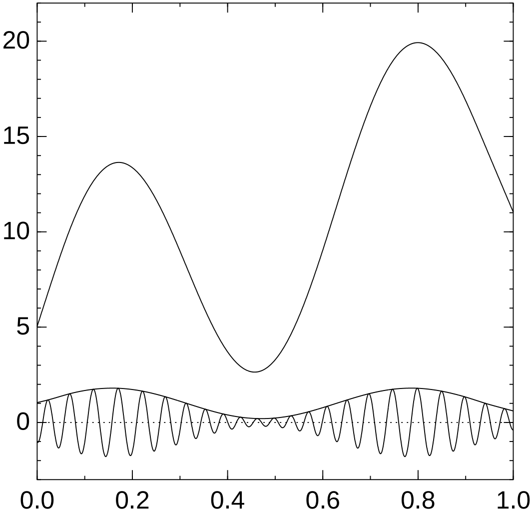
extracting the residual



calculate residual r_1 as mean of upper and lower envelope

IMF extraction

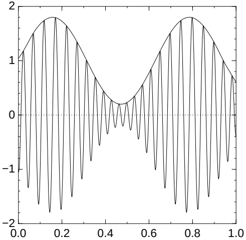
final signal separation



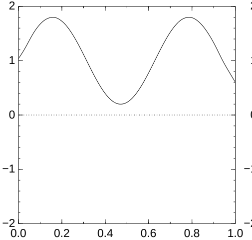
calculate IMF u_0 and a_0 by subtracting r_1 from s_1 and u. envelope

IMF extraction

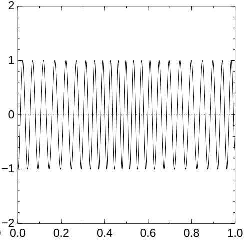
final IMF separation (1/2)



$u_0(t),\ a_0(t)$



$a_0(t)$

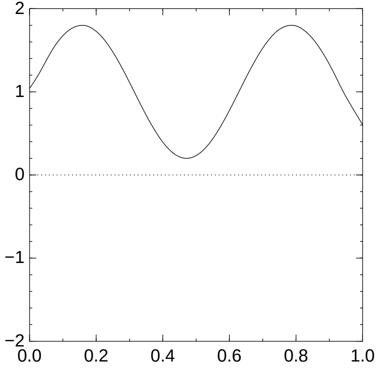


$u_0(t)/a_0(t)$

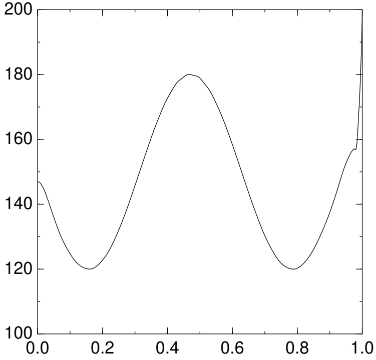
last step apply simple operator-based method to
$u_0(t)/a_0(t) = \cos(\phi(t))$

IMF extraction

final IMF separation (2/2)



$a_0(t)$



$\phi_0'(t)$

designation: hybrid operator-based methods

theoretical results

optimization problem for input signal s

$$\begin{aligned} \min_{(a,\phi)} \quad & \|a \cdot \cos(\phi) - s\| \\ \text{s.t.} \quad & a \cdot \cos(\phi) \text{ IMF.} \end{aligned}$$

- set of IMF 'souls' (a, ϕ) is convex
- cost function $\|a \cdot \cos(\phi) - s\|$ is convex-like
- optimization problem is SLATER-regular $\rightarrow$ strong duality

justification for heuristics of the form

$$\min_{(a,\phi)} \quad \|\text{cost}\| + \|\text{regularization operator}\|$$

numerical results

- C-toolbox
  - B-Spline precomputation
  - sparse linear (weighted) least-squares fits
  - plotting
  - complexity
- proposed envelope procedure converges

outlook

- substitute GSL for a more specific library
- find higher-order linear operators (or prove non-existence), as

$$\mathcal{D}^{G}_{(1,\phi)} f := \left(\frac{1}{(\phi')^2} \right) \cdot f'' + \frac{1}{2} \cdot \left(\frac{1}{(\phi')^2} \right)' \cdot f' + f$$

  only goes up to f''